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Classifying split extensions of quantum groups

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Joint work with: C.G. Bontea and G. Militaru

For two given Hopf algebras A and H we classify all Hopf algebras E that fit into a sequence $A \hookrightarrow E \to H$ such that $E \to H$ splits in the category of coalgebras and $A = E^{\mathrm{co}(H)}$. Equivalently, we classify all crossed products of Hopf algebras $A\#H$. The classification is up to an isomorphism of Hopf algebras that stabilizes A (resp. co-stabilizes H) by two relative cohomological 'groups' $\mathcal{H}^2_A(H, A)$ (resp. $\mathcal{H}^{2,H}(H, A)$) in the construction of which the role of coboundaries is played by pairs (r, v) consisting of a unitary cocentral map $r : H \to A$ and an automorphism v of H (resp. A). At the level of groups these objects are two different quotients of the classical second cohomology group. All crossed products $A\#H_4 := A_{(a\,|\,g,\,x)}$ are explicitly described by generators and relations and classified: these quantum groups $A_{(a\,|\,g,\,x)}$ are parameterized by the set $\mathcal{ZP}(A)$ of all central primitive elements of A. Several examples are worked out in detail: in particular, over a field of characteristic $p \geq 3$ an infinite family of non-isomorphic quantum groups of dimension $4p$ is constructed. For the cyclic group C_n, all crossed products $H_4\#k[C_n]$ are explicitly described and classified in four possible ways. They are $4n$-dimensional quantum groups $H_{4n,\,\lambda,\,t}$, associated to all pairs $(\lambda,\, t)$ consisting of an arbitrary unitary map $t : C_n \to C_2$ and an n-th root λ of ± 1 – the choice of the sign $\pm$ being dictated by the signature of t. The groups of automorphisms of these quantum groups are described.

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Representations of conformal Galilei algebra with integer spin and an application

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The conformal Galilei algebra (CGA) is a class of non-semisimple Lie algebras. Each member of this class is specified by two parameters (d, ℓ). The parameter d is a positive integer and the parameter ℓ takes an integer or a half-integer value.

In the present work we investigate the irreducibility of the lowest weight representations of CGA with $d = 1$ and any integer ℓ. We first show that all Verma modules are reducible by giving an explicit formula of the Kac determinant. Then a list of singular vectors in the Verma modules is presented. With the list, hierarchies of partial differential equations (defined on the coset space of the group generated by CGA) are derived. A list of all irreducible lowest weight modules of CGA is also obtained.

Matrix valued orthogonal polynomials related to the quantum analogue of $(\mathrm{SU}(2) \oplus \mathrm{SU}(2), \mathrm{diag})$

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Classically the scalar spherical functions on $\mathrm{SU}(2) \oplus \mathrm{SU}(2)$ invariant under the diagonal of $\mathrm{SU}(2) \oplus \mathrm{SU}(2)$ can be identified with the Chebychev polynomials of the second kind. In [1, 2] the spherical functions on the symmetric pair $(\mathrm{SU}(2) \oplus \mathrm{SU}(2), \mathrm{diag})$ were generalized to matrix valued spherical functions. The matrix valued spherical functions can be identified with matrix valued orthogonal polynomials and explicit differential equations, orthogonality relations and three term recurrence relations were found.

On this poster we quantize these matrix valued spherical functions to matrix valued spherical functions on the quantized universal enveloping algebra $\mathcal{U}_q = \mathcal{U}_q(\mathfrak{sl}_2) \otimes \mathcal{U}_q(\mathfrak{sl}_2)$ which are invariant under the action of the right coideal $\mathcal{B}$, which was found using the theory of Letzter [3]. The right coideal $\mathcal{B}$ can be seen as the quantization of the diagonal in $\mathrm{SU}(2) \oplus \mathrm{SU}(2)$. We identify the matrix valued spherical functions with matrix valued polynomials such that for each $\ell \in \frac{1}{2}\mathbb{N}_0$ we have a family $(P_n^\ell)_{n \geq 0}$ of matrix valued orthogonal polynomials of dimension $(2\ell + 1) \times (2\ell + 1)$. Using the group structure of $\mathcal{U}_q$ and the right coideal structure of $\mathcal{B}$ we find explicit two second order q-difference equations, orthogonality relations and the three term recurrence relation for the matrix valued orthogonal polynomials.

These derivations are interesting in their own right since they combine on the one hand the group structure and representation theory of the quantum group $\mathcal{U}_q$ and on the other hand the theory of special functions and (basic) hypergeometric series.

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On the real differential of a slice regular function

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In a recent work by G.Gentili, S.Salamon and C.Stoppato, the authors shows an interesting relation between the character of quaternionic functions of one quaternionic variable which are slice regular[1] and the complex geometry of $\mathbb{R}^4$ minus a parabola. The set of functions considered in their work is restricted to a certain subclass in which the domain of definition has nonempty intersection with the real axis. In this talk I will show that at least the analytic setting is restored if the domain of definition of such functions does not intersects the real axis.

[1]definitions and main properties will be given

Complexity Analysis of Sleep Based on Non-Invasive Techniques

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Joint work with: Philip Holloway, Alan St Clair Gibson, Jason Ellis

This work focuses on a method for investigation of physiological time series based on complexity analysis. It is part of a wider programme to determine non-invasive markers for healthy ageing. We consider the case study of sleep and alternations with insomnia. For the first time, fractal analysis techniques are implemented to study the correlations present in sleep actigraphy for individuals suffering from acute insomnia with comparisons made against healthy subjects. Analysis was carried out for 21 healthy individuals with no diagnosed sleep disorders and 26 subjects diagnosed with acute insomnia during night-time hours. Detrended fluctuation analysis was applied in order to look for $1/f$-fluctuations indicative of high complexity. The aim was to investigate if complexity analysis can differentiate between people who sleep normally and people who suffer from acute insomnia. The hypothesis was that the complexity will be higher in subjects who suffer from acute insomnia due to increased night time arousals. The complexity results for nearly all of the subjects fell within a $1/f$-range, indicating the presence of underlying control mechanisms. The subjects with acute insomnia displayed significantly higher levels of complexity, possibly a result of too much activity in the underlying regulatory systems. Moreover, we found a linear relation between complexity and variability, both of which increased with the onset of insomnia [1]. Fractal techniques have never been applied to actigraphy of sleep before with the hypothesis of high correlations being a marker of sleep-related health issues. This study showed that the complexity approach is very promising and could prove to be a useful non-invasive identifier for people who suffer from sleep disorders such as insomnia. Further examples will be demonstrated, showing that complexity analysis can also be used to investigate healthy ageing, and a number of open problems will be discussed.

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On the raising and lowering difference operators for eigenvectors of the finite Fourier transform

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We construct explicit forms of raising and lowering difference operators that govern eigenvectors of the finite (discrete) Fourier transform. Some of the algebraic properties of these operators are also examined.

Critical Behaviour in Semi-classical Matrix Models and Painleve-Type Equations

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We consider a singular perturbation to an invariant unitary ensemble for which the measure on $N \times N$ hermitian matrices takes the form,

$$d\mu_k(M) := \frac{1}{Z_N} e^{-N\left[V(M)+\left(\frac{s}{M}\right)^k\right]} dM.$$

Here the potential $V(x)$ is polynomial and Z_N is the normalisation constant. This model with $k = 1$ has been recently studied by a number of authors, due to both its practical applications and its purely mathematical properties. For example, a matrix integral of this form has relevance to spin glasses, Akemann et al. (2014) and number theory, Brightmore et al. (2012).

Of particular interest has been the double scaling limit obtained by allowing the strength of the perturbation s to go to zero as the size of the matrix is taken to infinity. In this presentation we review the work of Xu et al. (2013) and Brightmore et al. (2012) and then present a generalisation of their results to the case when k, the order of the singular perturbation, is increased. We then discuss how the structures found in this case generalise to arbitrary semi-classical matrix models in which the derivative of the potential is rational and semi-infinite integration contours are allowed.

Classical Yang-Baxter equation and some related structures

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In this talk, I briefly introduce what is the classical Yang-Baxter equation with emphasizing the unification of the tensor and the operator forms of the classical Yang-Baxter equation. As applications, certain generalizations of the classical Yang-Baxter equation with motivation from the study of integrable systems and some new algebraic structures with an operadic interpretation are given. Furthermore, some bialgebraic structures are constructed upon the relationship between Lie bialgebras and the classical Yang-Baxter equation.

Three dimensional gravity, holography and Virasoro coadjoint orbits

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The symmetry group of three dimensional asymptotically anti-de Sitter spacetimes is the conformal group in two dimensions. In the asymptotically flat case, it is the BMS group in three dimensions, which is the semi-direct product of the diffeomorphism group on the circle with its adjoint representation embedded as an abelian normal subgroup. We show that the reduced phase space of these three dimensional gravity theories is completely described by the coadjoint representation of their symmetry groups. As a consequence, a series of natural questions in 3d gravity, such as positivity of energy for instance, can be phrased and answered in terms of the well-known classification of Virasoro coadjoints orbits and their properties.

Strong contraction of the Discrete Series representations of SU(1,1) via the Kirillov model

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Joint work with: Eyal Subag, Joseph Birman, Ady Mann

We describe the strong contraction of the discrete series representations of SU(1,1) to the irreducible unitary representations of Iso(1,1). The contraction is obtained via the Kirillov model. We describe the smooth space of the Kirillov model and the action of the Weyl element of SU(1,1) as an Hankel transform.

Fundamental group of a monstrous(?) discriminant complement

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Joint work with: Daniel Allcock

I want to talk about a curious arithmetic subgroup Γ of the rank one real Lie group $U(1, 13)$. The group Γ acts on the unit ball $\mathbb{B}$ in $\mathbb{C}^{13}$ (the hermitian symmetric space of $U(1, 13)$, also known as the 13 dimensional complex hyperbolic space). Let $\mathcal{H}$ be the co-dimension one strata of the set of points in $\mathbb{B}$, that have non-trivial stabilizer under Γ action. A conjecture due to Daniel Allcock (see [1]), predicts that the fundamental group of the orbifold $(\mathbb{B} \setminus \mathcal{H})/\Gamma$ maps onto the semi-direct product of $M \times M$ and $\mathbb{Z}/2\mathbb{Z}$ (where M is the Monster simple group). Proof of this conjecture would yield a complex 13-dimensional orbifold with a natural action of $(M \times M) \rtimes (\mathbb{Z}/2\mathbb{Z})$.

I shall describe the rationale behind the conjecture (driven by analogies with Weyl groups), the evidences for it, and recent progress towards its proof. The Leech lattice plays a major role in our construction of Γ as well as in our arguments. Below we give some references. More complete bibliography are in these articles.

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On equations of motion on Siegel-Jacobi spaces generated by linear Hamiltonians in the generators of the Jacobi group

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The Jacobi group is the semidirect product of the Heisenberg group with the symplectic group. The Siegel-Jacobi spaces are homogeneous Kähler manifold associated to the Jacobi group and they consist as sets with the product of complex space of dimension n with the Siegel ball of the same index. We consider the classical and quantum motion generated by a Hamiltonian linear in the generators of the Jacobi group. We show that the equations of motion obtained via Berezin’s recipe are identical with the equations of motion obtained with Wei-Norman procedure.

Fractional powers of Riesz transforms and applications

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An important tool in image feature detection are monogenic signals which are monogenic functions in the upper half space $\mathbb{R}^3_+$. The monogenic signal of a scalar valued signal $f = f(x_1, x_2)$ is

$$f_M(x_1, x_2) = f(x_1, x_2) + \mathcal{R}_1 f(x_1, x_2) + \mathcal{R}_2 f(x_1, x_2),$$

where $\mathcal{R}_j$ are the Riesz operators with Fourier transform

$$\widehat{\mathcal{R}_j f} = i\frac{\xi_j e_j}{|\xi|}\hat{f},$$

and e_j are Clifford generating vectors.

A basic problem in image processing is the detection of edges and wedges. It is well known that such enhancements can be done by second order derivatives. Because the Riesz operators work like derivatives one may ask if powers of Riesz operators work in the same way like higher order derivatives. Therefore we consider powers and also fractional powers of a certain type of Riesz operators:

$$(\mathcal{R}_1 - e_1 e_2 \mathcal{R}_2)^\alpha\,,\ \alpha > 0, \alpha \in \mathbb{R}.$$

It turns out that such an operator can be linked to fractional Dirac operators and to spherical harmonics if $\alpha = n \in \mathbb{N}$. That Riesz-type operator can also be realized in an optical setting for image enhancement.

The Schwarz-Pick Lemma for slice regular functions.

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Joint work with: C. Stoppato

Abstract

The celebrated Schwarz-Pick lemma for the complex unit disk is the basis for the study of hyperbolic geometry in one and in several complex variables. In the present talk, we turn our attention to the quaternionic unit ball $\mathbb{B}$. We prove a version of the Schwarz-Pick lemma for self-maps of $\mathbb{B}$ that are slice regular, according to the definition of Gentili and Struppa. The lemma has interesting applications in the fixed-point case.

An integrable Hénon-Heiles system on the sphere and the hyperbolic plane

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The integrable Hénon–Heiles systems can be written as particular cases of the multiparametric family of two-dimensional Hamiltonian systems given by

$$\mathcal{H} = \frac{1}{2}(p_1^2 + p_2^2) + \Omega_1\, q_1^2 + \Omega_2\, q_2^2 + \alpha\left(q_1^2 q_2 + \beta\, q_2^3\right),$$

where Ω_1, Ω_2, α and β are real constants. In particular, the only known integrable cases are: (i) the Sawada–Kotera Hamiltonian ($\beta = 1/3,\ \Omega_1 = \Omega_2$), (ii) the Korteweg–de Vries (KdV) one ($\beta = 2$), and (iii) the Kaup–Kupershmidt system ($\beta = 16/3, \Omega_2 = 16\,\Omega_1$).

In this contribution we present a constant curvature analogue $\mathcal{H}_\kappa$ of the integrable Euclidean Hénon–Heiles KdV Hamiltonian, whose construction is carried out once the corresponding curved Ramani potentials are previously obtained.

This framework is based on the use of the Gaussian curvature of the underlying spaces as an explicit deformation parameter κ, thus connecting the Euclidean and the curved systems (sphere/hyperbolic space) in a smooth way, and presenting the integrable curved KdV Hamiltonian in a geometric unified approach. In particular, the Euclidean case is obtained as the flat limit $\kappa \to 0$ performed over the new integrable system $\mathcal{H}_\kappa$.

Boundary Conditions, Semigroups and the Quantum Arrow of Time

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Dynamical equation for quantum physics are solved under Boundary Conditions which specify the space of admitted solutions. The Hilbert space boundary condition, as well as the Schwartz space of Dirac's formalism, lead by mathematical theorems to (unitary) group evolution with $-\infty < t < +\infty$. But heuristic arguments (causality) require a beginning of time $t_0 < t < +\infty$. This t_0 is observed as an ensemble of finite beginning of time $\{t_0^{(i)}\} \leftrightarrow t_0$ of single quantum systems (like the onset time $\{t_0^{(i)}\}$ of the i-th dark period in Dehmelt's quantum jump experiments on single ion). This t_0 suggests time asymmetric boundary conditions, which are provided by Hardy spaces boundary condition (e.g, in the Lax-Phillips theory). The Paley-Wiener theorem for Hardy spaces using these boundary conditions for the solutions for the dynamical equations of quantum theory, then leads to quantum mechanical semi-group evolutions expressing time asymmetry in quantum physics.

Spin-orbit coupling in low-symmetry systems

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A quantised state is most formally labelled by the irreducible representation of the group which describes its symmetry together with a number which simply indicates the energetic order of the states described by the same representation. In high-symmetry systems, especially those for which the Schrödinger equation is solvable, one or more quantum numbers may be defined which have a relationship to one or more of the characters of the irreducible representation. Subduction to lower symmetry groups may provide one or more component quantum numbers and this approach is particularly useful in the presence of a homogeneous magnetic field. Little attention has been paid to the effect automorphisms of the symmetry group have on the characterisation of the physical state by either symmetry labels or by quantum numbers. These problems will be addressed with spectroscopic examples illustrating both point groups and space groups.

Relativistic Resonance and Decay Phenomena

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Joint work with: Arno Bohm

The relation $\tau = \hbar/\Gamma$ between the width of a resonance Γ and the lifetime τ of a decaying state associated with the resonance in the relativistic domain have been accepted from non-relativistic quantum theory, which was approximately obtained in Weisskopf-Wigner methods in the Hilbert space. In order to obtain the exact relation, one has to modify Hilbert space axiom, in which the space representing the states and the space representing the observable are indistinguishably, to new boundary conditions named the Hardy space axioms in which the space of the states and the space of the observables are described by two distinguishable spaces. As a result of the new boundary conditions, instead of having the symmetrical time evolution for the state and the observables described by the unitary groups which are the consequence of the Hilbert space axioms, one gets the asymmetrical time evolutions for the states and observables described by the semi-groups. The relativistic resonance can be described by the relativistic Gamow vector, which is defined as superposition of the exact out-plane waves with the Breit-Winger energy distribution and obeys the exponential law.

Special functions, Lie algebras and Quantum Mechanics

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Special functions are introduced as transformation matrices between discrete and continuous bases in quantum mechanics. The unitary irreducible representation of a Lie algebra to which each special function belongs, allows indeed a formal description where a rigged Hilbert space is naturally present.

Each element of the corresponding Lie group defines, starting from a standard basis, a new basis that can be continuous, discrete or mixed. The Lie structure determines also the space of the linear operators acting on the quantum mechanics space that is found to be homomorphic to the one of the considered Universal Enveloping Algebra.

The line and the half line with their 3 alternative standard bases (2 continuous and 1 discrete) are discussed in detail in connection with Hermite and Laguerre functions.

Riemann-Hilbert problems for discrete monogenic functions over the half space

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In the last decade there is an increased interest in constructing discrete counterparts to well-known continuous function theories. This is mainly motivated by recent results of S. Smirnov which connect complex discrete function theory with problems in probability and statistical physics. But while such ideas are quite well developed in the complex case the higher-dimensional case is yet rather underdeveloped. This is due to the fact that discrete Clifford analysis started effectively only in the eighties and nineties with the construction of discrete Dirac operators either for numerical methods of partial differential equations or quantized problems in physics. One of the important applications is in the study of Riemann-Hilbert problems. Unfortunately, here a principal problem arises. Continuous and discrete boundaries and boundary values behave quite differently. In this talk we will highlight the differences and present the definition of discrete Hardy spaces. Afterwards we discuss their link with boundary values of discrete monogenic functions and apply it to the study of discrete Riemann-Hilbert boundary value problems for the half space.

KP hierarchy for a cyclic quiver

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Joint work with: Alexey Silantyev

We introduce a generalisation of the KP hierarchy, which is intimately related to the cyclic quiver with m vertices; the case $m = 1$ corresponds to the usual KP hierarchy. Generalising the result of [1], we show that our hierarchy admits special solutions parameterised by suitable quiver varieties. Using the approach of [2], we identify the dynamics of the singularities for these solutions with the classical Calogero–Moser system for the complex reflection groups $G(m, 1, n)$. The link to the bispectral operators from the work [3] will be indicated.

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Extensions of natural Hamiltonians

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Joint work with: Luca Degiovanni, Giovanni Rastelli

Extension of natural Hamiltonians can be seen as a particular way to make two Hamiltonians interacting, in such a way that an additional non-trivial polynomial in the momenta first integral is recursively computed, of degree depending on a integer or rational parameter.

Different aspects of this technique are discussed, as well as the deep link with warped manifolds and the possibility of constructing symmetry operators for the corresponding quantum systems.

Some recent improvements in the effectiveness of the method are presented.

Entanglement, Entropy and Representation Theory

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Entanglement and entropy are key concepts in quantum information theory. In this talk, I will explain what we can learn about these two with today's representation theory of the symmetric and unitary groups. As I will show, open problems from the study of entanglement and entropy point to a possible future direction in representation theory.

Spectral properties of quaternionic linear operators

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Joint work with: D. Alpay, D.P. Kimsey, I. Sabadini

We show that the S-spectrum, introduced for the quaternionic version of the S-functional calculus, is the spectrum that must be used in the quaternionic spectral theorem for unitary operators. In the case of quaternionic normal matrices the proof of the spectral theorem is based on the notion of right-spectrum, which coincide with the S-spectrum for all quaternionic matrices. Our results restore the analogy with the complex case in which the classical notion of spectrum appears in the Riesz-Dunford functional calculus as well as in the spectral theorem.

Homological properties of category O

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Joint work with: V. Mazorchuk and V. Serganova

In the first part we study homological properties of category $\mathcal{O}$ and the related category of Harish-Chandra bimodules for semisimple Lie algebras. The main theme is the concept of extension fullness. We prove extension fullness of (thick) category $\mathcal{O}$ in the category of (generalised) weight modules and extension fullness of thick category $\mathcal{O}$ in the category of all modules. Then we focus on the weak Alexandru conjectures; we prove this conjecture for regular blocks of (thick) category $\mathcal{O}$, but disprove it for singular blocks. As an extra result we obtain homological dimensions of structural modules in singular blocks, which will be applied in the second part.

In the second part we focus on homological invariants of category $\mathcal{O}$ for basic classical Lie superalgebras. We demonstrate the relation between finiteness of projective dimensions, relative homological algebra and the associated variety. We calculate projective dimensions of the structural modules and introduce the concept of complexity to study modules with infinite projective dimension. Finally we apply these results in the study of equivalences between different blocks in category $\mathcal{O}$.

On a Microscopic Representation of Spacetime

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Using the Dirac algebra γ^μ as initial stage of our discussion, we find the isomorphic 15dimensional Lie algebra su*(4) as complex embedding of sl(2,**H**). Based on previous work, we can identify spin-isospin hadron states as representations of the compact group SU(4), and it is very interesting to discuss various group chains based on SU*(4) or SU(4) resp. their maximal compact subgroup USp(4). Within this context, we discuss some exponential representations emerging in coset decompositions, some of their properties and their relation to 'wellknown' procedures of standard QFT. On the other hand, these technical procedures hide a lot of the geometrical (and physical) background. By reverting to very old (and at that time basic) knowledge of physics and mathematics, at first we are led to line geometry and some transfer principles which technically reflect in the group chains found above by choosing Lie's (differential) approach. The geometrical description, however, leads to the beautiful framework of line complexes which comprizes Dirac's 'square root of p^2', the discussion of 'equations of motion' mostly in terms of 'moving points', electromagnetism, etc. This old framework not only covers these actual approaches but it also restores simple access to physical properties in terms of polar and null polar systems after having introduced homogeneous coordinates and projective geometry.

Integrable Dicke and Jaynes-Cummings models, and extensions

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Joint work with: Pieter Claeys and Dimitri Van Neck

The interaction of atomic or molecular states with quanta of electromagnetic radiations is described by the Jaynes-Cummings Hamiltonian, and its generalization the Dicke Hamiltonian. It has been shown that these models are integrable by means of a Bethe Ansatz state, coupling the electromagnetic quanta with the excitation modes of the atoms. In this contribution, it will be discussed how the Dicke model can be derived from the hyperbolic Richardson-Gaudin integrable models by means of a pseudo-deformation of the quasispin algebra, and several extensions will be presented

Affine symmetry principles for non-crystallographic systems and applications to viruses and carbon onions

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Joint work with: Reidun Twarock, Celine Boehm, Jess Wardman, Tom Keef

We explore the possibility that extended icosahedral systems occurring in nature may be describable by an extension of the icosahedral group by a non-compact operation. We derive such extensions in two ways, firstly by direct Kac-Moody-type extension of the (non-crystallographic) H_3 root system, and secondly by projection of the affine (crystallographic) D_6 root system. We discuss applications to the structure of viruses as well as an affine icosahedral construction of nested fullerene shells (so-called carbon onions), suggesting that affine symmetry occurs more widely in nature.

Approximation properties for quantum groups

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Joint work with: A. Freslon and M. Yamashita

Several notions of group theory (such as amenability, weak amenability and the Haagerup property) continue to make sense for quantum groups. When the approximation property (A) satisfies a certain centrality condition (automatically satisfied in the case of discrete groups), one says that the quantum group has central (A). In this talk, we will consider approximation properties for the dual of the quantum group $SU_q(2)$ (where $0 < q < 1$). It is well-known that this dual is amenable but not centrally amenable. We will show that, nevertheless, it is centrally weakly amenable. As a corollary, we obtain that also the free discrete quantum groups $\mathbb{F}O_F$ and $\mathbb{F}U_F$ of Van Daele and Wang are centrally weakly amenable for any matrix F.

Quantum supergroups

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In this talk, we introduce topological quantum supergroups and their representations. Then, we expose the construction of the non-formal deformation quantization of the abelian supergroups $\mathbb{R}^{m|n}$ and its universal deformation formula. By using it, we construct a class of Fréchet solvable quantum supergroups as well as quantum supergroups with supertoral subgroups. For such quantum supergroups, we find an analog of Kac-Takesaki operators that are superunitary and satisfy the pentagonal relation.

Landscape symmetries in three-photon coincidence landscapes

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Joint work with: Barry C. Sanders, Si-Hui Tan and Isaac P. Poulin

We discuss how the permutation group and group-theoretical methods can be used to analyze features of the coincidence rate for three photons exiting in three distinct ports of an interferometer as a function of relative time delays (τ_1, τ_2) between these photons. By delaying the times-of-arrival of these photons, which are otherwise identical, partial distinguishability is controllable, and landscapes of coincidence rates vs delay times between pairs of photons are explained in terms of immanants or group functions.

For monochromatic photons, we describe the simple connection between the coincidence rate and immanants of the scattering matrix, and the covariance properties of the rates. For pulse inputs, we suggest the analysis is simpler when done in terms of group functions because of some covariance properties of the final expressions.

Although the focus will be primarily three photons in a three-port interferometer, we will also point out how the same techniques can be used for three-photon passive interferometers with more channels.

A Hamilton-Jacobi theory for Classical Field Theories: finite and infinite dimensional settings

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We will present a Hamilton-Jacobi theory in the framework of multisymplectic manifolds, including the corresponding theory on the infinite dimensional counterpart of the space of Cauchy data.

Constant surfaces associated to supersymmetric $\mathbb{C}P^{N-1}$ sigma models

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Joint work with: Véronique Hussin, Ismeth Yurdusen and Wojtek J. Zakrzewski

In this talk, I will consider constant curvature surfaces obtained from the classical solutions of the supersymmetric $\mathbb{C}P^{N-1}$ sigma models. These surfaces are obtained from the Weierstrass immersion formulae and it will be shown that these surfaces live in the Lie algebra $su(N)$. Furthermore, explicit non-holomorphic solutions of the model are considered for the first time and I'll give the different strategies that we have used to obtain such solutions. These strategies are based on a novel approach using a gauge-invariant formulation of the model in terms of orthogonal projectors.

Lie groups, Jacobi polynomials and Wigner d-matrices

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Joint work with: Enrico Celeghini and Miguel Angel Velasco

A $SU(2,2)$ symmetry group for Jacobi polynomials $J_n^{(\alpha,\beta)}(x)$and Wigner d_j-matrices is obtained in terms of ladder operators for integer and half-integer values of $j = n + ()/2$. Jacobi polynomials and Wigner d_j-matrices support a unitary irreducible representation of $SU(2,2)$ characterized by the eigenvalue of the quadratic Casimir $\mathcal{C}_{SU(2,2)} = -3/2$. We are able to construct different group structures ($SU(1,1)$, $SO(3,2)$, $Spin(3,2)$), whose representations separate integers and half-integers values of j, from the Universal Enveloping Algebra of $su(2,2)$. Moreover appropriate L^2–function spaces are realized inside the support spaces of all these representations and the operators acting on these L^2–function spaces belong to the corresponding Universal Enveloping Algebra.

Orthogonal polynomials and dressing chains

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Joint work with: S. Tsujimoto, L. Vinet, and A. Zhedanov

At first we will consider a generic notion of Darboux transformations for tridiagonal matrices. In particular, Darboux transformations for tridiagonal linear pencils in two variables will be presented. Then we will see that dressing chains for Askey-Wilson polynomials and polynomials from q=-1 world lead to quadratic algebras.

New results on the radially deformed Dirac operator

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Joint work with: Hendrik De Bie and David Eelbode

Let $\{e_i\}, i = 1, \ldots, m$, be an orthonormal basis of $\mathbb{R}^m$. The orthogonal Clifford algebra $\mathcal{C}l_m$ is generated by this basis under the relations $e_ie_j + e_je_i = -2\delta_{ij}$. It is well-known that the classical Dirac operator $\partial_{\underline{x}} = \sum_{i=1}^{m} e_i\partial_{x_i}$ and its Fourier symbol $\underline{x} = \sum_{i=1}^{m} e_i x_i$ generate via Clifford multiplication a natural Lie superalgebra $\mathfrak{osp}(1|2)$ contained in the Clifford-Weyl algebra. More surprisingly, this carries over to a natural family of deformations of the Dirac operator. Indeed, the radially deformed Dirac operator

$$\mathbf{D} := \partial_{\underline{x}} + c|\underline{x}|^{-2}\underline{x}\mathbb{E}, \qquad c > -1$$

and the vector variable $\underline{x}$ also form a realization of $\mathfrak{osp}(1|2)$ in the Clifford-Weyl algebra. Here $\mathbb{E} = \sum_{i=1}^{m} x_i\partial_{x_i}$ is the Euler operator measuring the degree of homogeneity.

In [H. De Bie, B. Ørsted, P. Somberg and V. Souček, Dunkl operators and a family of realizations of $\mathfrak{osp}(1|2)$. *Trans. Amer. Math. Soc.* **364** (2012), 3875–3902] several function theoretical aspects of the operator $\mathbf{D}$ are studied, such as the associated measure, the Fischer decomposition, the related Laguerre polynomials and the associated Fourier transform defined by

$$\mathcal{F}_D = e^{i\frac{\pi}{2}\left(\frac{1}{2}+\frac{m-1}{2(1+c)}\right)} e^{\frac{-i\pi}{4(1+c)^2}\left(D^2-(1+c)^2\underline{x}^2\right)}.$$

The investigation of this radially deformed Fourier transform is continued in [H. De Bie, B. Ørsted, P. Somberg and V. Souček, The Clifford deformation of the Hermite semigroup. *SIGMA 9* (2013), 010, 22 pages], using a group theoretical approach.

The aim of this talk is to further develop the study of the radially deformed Dirac operator, focussing on the conformal structure of $\mathbf{D}$ and the derivation of an explicit form for the kernel of the deformed Fourier transform $\mathcal{F}_D$.
We construct a set of differential operators that leave the kernel of $\mathbf{D}$ invariant. These set of generalised symmetries of $\mathbf{D}$ generate a Lie algebra which is isomorphic to the conformal Lie algebra $\mathfrak{so}(m + 1, 1)$. Next, a version of Stokes' theorem for the radially deformed Dirac operator is obtained. Moreover, explicit formulas for the kernel of the radially deformed Fourier transform $\mathcal{F}_D$ are obtained when the dimension is even and $1 + c = \frac{1}{n}$, $n \in \mathbb{N}_0 \setminus \{1\}$ with n odd. Finally, the realization of $\mathfrak{osp}(1|2)$ is generalized by adding suitable odd powers of $\Gamma = -\underline{x}\partial_{\underline{x}} - \mathbb{E}$ to the radially deformed Dirac operator.

Poisson vertex algebras and Hamiltonian equations

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Joint work with: Victor Kac

Poisson vertex algebras play a central role in the theory of integrable systems. After a brief introduction to the theory, we study the integrability of bi-Hamiltonian systems with a compatible pair of local Poisson structures arising in the study of cohomology of moduli spaces of curves.

On two subgroups of U(n), important for quantum computing

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Joint work with: Stijn De Baerdemacker

As two basic building blocks for any quantum circuit, we consider the 1-qubit `PHASOR` circuit and the 1-qubit `NEGATOR` circuit:

$$\texttt{PHASOR}(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \text{ and } \texttt{NEGATOR}(\theta) = \begin{pmatrix} \cos(\theta)e^{-i\theta} & i\sin(\theta)e^{-i\theta} \\ i\sin(\theta)e^{-i\theta} & \cos(\theta)e^{-i\theta} \end{pmatrix} .$$

Both are roots of the `IDENTITY` circuit. Indeed: $\texttt{PHASOR}(0) = \texttt{NEGATOR}(0) = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$. Additionally, the `NEGATOR` is a root of the `NOT` gate. Indeed: $\texttt{NEGATOR}(\pi) = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. Quantum circuits (acting on w qubits) consisting of controlled `PHASOR`s are represented by matrices from ZU(2^w); quantum circuits consisting of controlled `NEGATOR`s are represented by matrices from XU(2^w). Here, ZU(n) and XU(n) are subgroups of the unitary group U(n):

- the group XU(n) consists of all $n \times n$ unitary matrices with all $2n$ line sums (i.e. all n row sums and all n column sums) equal to 1 and
- the group ZU(n) consists of all $n \times n$ unitary diagonal matrices with first entry equal to 1.

ZU(n) is isomorphic to U$(1)^{n-1}$ and XU(n) is isomorphic to U($n-1$). We conjecture that any U(n) matrix can be decomposed into four parts:

$$U = \exp(i\alpha)\, Z_1 X Z_2 \ ,$$

where both Z_1 and Z_2 are ZU(n) matrices and X is an XU(n) matrix. We give a 'Sinkhorn-like' algorithm to find the decomposition. For $n = 2^w$, it leads to a synthesis algorithm for an arbitrary quantum computer.

A. De Vos and S. De Baerdemacker:
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[2] "Logics between classical logic and quantum logic", *9 th Int. Workshop on Quantum Physics and Logic*, Bruxelles, 10 - 12 Oct. 2012, pp. 123 - 128.
[3] "The `NEGATOR` as a basic building block for quantum circuits", *Open Systems & Information Dynamics* **20** (2013), 1350004.
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Supersymmetric harmonic oscillator and nonlinear coherent states

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Joint work with: Dr. David Jose Fernandez Cabrera

We find and analyze the supercoherent states associated with the generalized nonlinear supersymmetric annihilation operator (SAO) of a quantum harmonic oscillator with spin $s = 1/2$, also called SUSY harmonic oscillator. We discuss as well the uncertainty relation $\sigma_x^2 \sigma_p^2$ for a special annihilation operator in order to compare our results with those obtained for the linear supercoherent states.

The equivalence of Drinfeld-Sokolov, bihamiltonian and Dirac reductions

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We prove that the classical W-algebra associated to a nilpotent orbit in a simple Lie-algebra can be constructed by preforming bihamiltonian, Drinfeld-Sokolov or Dirac reductions. We conclude that the classical W-algebra depends only on the nilpotent orbit but not on the choice of a good grading or an isotropic subspace. In addition, using this result we prove again that the transversal Poisson structure to a nilpotent orbit is polynomial and we better clarify the relation between classical and finite W-algebras.

Invariant Differential Operators for Non-Compact Lie Algebras Parabolically Related to Conformal Lie Algebras

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In the present paper we review our project of systematic construction of invariant differential operators for non-compact semisimple Lie groups. Our starting points was the class of algebras, which we call 'conformal Lie algebras', which have very similar properties to the conformal algebras of Minkowski space-time, though we go beyond this class in a natural way. For this we introduce the new notion of *parabolic relation* between two non-compact semisimple Lie algebras $\mathcal{G}$ and $\mathcal{G}'$ that have the same complexification and possess maximal parabolic subalgebras with the same complexification.

Levi decomposable algebras in the classical Lie algebras

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Joint work with: J. Repka

We examine embeddings of an important class of Levi decomposable algebras into the classical Lie algebras. In particular, we classify the embeddings of $A_n \ltimes \mathbb{C}^{n+1}$, $B_n \ltimes \mathbb{C}^{2n+1}$, $C_n \ltimes \mathbb{C}^{2n}$, and $D_n \ltimes \mathbb{C}^{2n}$ into the complex simple Lie algebras A_{n+1}, B_{n+1}, C_{n+1}, and D_{n+1}, respectively, up to inner automorphism.

Energy-Momentum Tensor in Electromagnetic Theory and Gravitation from Relativistic Quantum Equations.

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We analize the problems of definitions of the energy-momentum tensor in the electromagnetic theory and general relativity. We compare our results with those which have been obtained by Acad. Logunov and Prof. Khrapko in the Minkowski space. The conclusion is that it is possible to define the energy-momentum tensors which are compatible with the Noether theorem in any relativistic theory within the Poincare group theory, in the consistent way. Different opinions, which can be found in the old literature, have been discussed. Possible methods for experimental check have been proposed.

Asymptotic behaviour of Lieandric numbers

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Joint work with: M. Jibladze

Lieandric numbers count the number of biparabolic Lie subalgebras of index 1 in full matrix algebras (these are examples of Frobenius Lie algebras, providing constant solutions of the Yang-Baxter equations). In the talk, a combinatoric description of these numbers will be given, a conjecture concerning their asymptotics will be formulated and some evidence for the conjecture will be presented.

Cauchy Type of Theorem in Hyperbolic Function Theory

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Joint work with: Heikki Orelma

Our aim is to build a function theory based on hyperbolic metric of the Poincaré upper half space model and Clifford numbers. We consider harmonic functions with respect to the Laplace-Beltrami operator of the Riemannian metric $ds^2 = x_2^{-\frac{2k}{n-1}} \left(\sum_{i=0}^{n} dx_i^2\right)$ and their function theory in $\mathbb{R}^{n+1}$. H. Leutwiler in 1992 discovered that the power function, calculated using the Clifford product, is a conjugate gradient of a harmonic function with respect to the hyperbolic metric. He started to research these type of functions, called H-solutions, satisfying a modified Dirac equation, connected to the hyperbolic metric. All usual trigonometric and exponential functions have a natural extensions to H-solutions. S.-L. Eriksson and H. Leutwiler, extended H-solutions to total algebra valued functions, called hypermonogenic functions.

We study generalized hypermonogenic functions, called $k-$hypermonogenic functions. For example the function $|x|^{k-n+1} x^{-1}$ is k-hypermonogenic. Note that 0-hypermonogenic are monogenic and $n-1$-hypermonogenic functions are hypermonogenic

We verify the Cauchy type integral formulas for k-hypermonogenic functions where the kernels are calculated using the hyperbolic distance of the Poincaré upper half space model. There formulas are different in even and odd cases. Earlier these results have been proved for hypermonogenic functions. The coauthor of the work is Heikki Orelma, Tampere University of Technology.

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q-Lie algebras and q-differential geometry united by examples

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The purpose of this talk is to present a unified theory for q-Lie algebras based on the usual Lie algebras with a modified q-addition. The objects called q-Lie groups are in fact manifolds with boundary. In all cases we use the matrix q-exponential to form the corresponding $n \times n$ matrix with q-determinant 1. When this is a q-Lie group, the corresponding matrix multiplication is twisted under the group morphism τ, which is intimately connected to the corresponding q-differential geometry. We can talk about q-analogues of the general linear group. Examples of the object $SU_q(n)$ as well as general q-spherical coordinates will be given. We show some Mathematica pictures of the most important objects.

As in the ordinary case, the q-Lie algebras are decomposed into compact matrix generators and noncompact matrix generators. The definitions of the corresponding q-symmetric spaces will have to wait until later, but examples can already be given.

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A construction of generalized Lotka-Volterra systems connected with simple Lie algebras

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Joint work with: Pantelis Damianou; Stelios Charalambides

In this talk I will describe an algorithm for producing Hamiltonian systems by constructing the corresponding Lax pairs. This is achieved by considering a larger subset of the positive roots than the simple roots of a simple complex Lie algebra. In several cases these subsets of the positive roots recover well known integrable Hamiltonian systems which are of Lotka-Volterra type. We call the systems produced by this method generalized Lotka-Volterra systems. We find a sufficient condition on the subsets of the positive roots of the root system of type A_n which give consistent Lax pairs and we classify the subsets of the positive roots of the simple Lie algebra of type A_n which produce, after a suitable change of variables, Lotka-Volterra systems. We also describe a variation of the algorithm in which we use complex coefficients. We show that in this case we produce more Lotka-Volterra systems.

Hidden Symmetries in Discrete Clifford Analysis

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The aim of this talk is mainly devoted to the construction of finite difference discretization schemes for the quantum harmonic oscillator $\mathcal{H} = -\frac{\hbar^2}{2m}\Delta + V(x)$ based on the factorization scheme, roughly speaking, an hypercomplex extension of Deift's approach for a general ODE considered in [2, Section 4].

Motivated by Cooper-Khare-Sukhature's approach for the Dirac equation in *continuum* (cf. [1, Section 11]), such approach is based on the construction of a pair of Clifford-vector-valued ladder operators $(A_h, A_h^\dagger)$ embody in a Clifford algebra of signature $(0, n)$ that involve finite difference discretizations of the *continuum* Dirac operator.

Special emphasize will be given for the construction of polynomial solutions based on Weyl-Heisenberg, $\mathfrak{su}(1,1)$ and $\mathfrak{osp}(1|2)$ symmetries (cf. [3, 4]) and for the exact solvability of the discrete model on the lattice $h\mathbb{Z}^n$ in the presence of symmetrized potentials $V_h(x)$ that converge to $V(x)$.

Despite the the *Lie dimensional reduction* of the number of ladder operators from $2n$ to 2 encoded by the hypercomplex extension of the standard factorization scheme for a chain of $n-$independent Hamiltonian operators, the symmetry breaking in lattice models that remain the Weyl-Heisenberg symmetries in the *continuum* limit will also be discussed along the talk.

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Recent developments in the reduction approach to action-angle dualities of integrable many-body systems

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Important integrable Hamiltonian systems can be often represented as reductions of "canonical free systems" having rich symmetries on higher dimensional phase spaces. The upstairs spaces include for example cotangent bundles of simple Lie groups together with their Poisson-Lie and quasi-Hamiltonian generalizations. Key properties of the reduced systems can be traced back to the original free systems. We first summarize the results of the last few years concerning the application of this reduction approach towards explaining the known action-angle duality relations between one-dimensional classical integrable many-body systems in group-theoretic terms. We then present fresh results such as the construction of new self-dual compact forms of the trigonometric Ruijsenaars-Schneider system and the description of a novel dual pair involving the trigonometric BC(n) Sutherland system. Finally, we give a brief account of the main open problems of the subject. The report will contain new as well as published results, for the latter see e.g. the joint paper by L.F. and T.J. Kluck in Nucl. Phys. B 882 (2014) 97-127, arXiv:1312.0400 and references therein.

Villamayor-Zelinsky sequence for braided finite tensor categories

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The classical Crossed Product Theorem states that the relative Brauer group with respect to a Galois field extension is isomorphic to the second Galois cohomology group. In 1977 Villamayor and Zelinsky introduced a cohomology for an extension of commutative rings and constructed an infinite exact sequence involving the respective cohomology groups. These groups are evaluated at three types of coefficients and the three types of cohomology groups appear periodically in the sequence. If the ring extension is faithfully flat, the relative Brauer group embeds into the middle term on the second level of the sequence. This sequence generalizes the Crossed Product Theorem to the case of commutative rings.

In 2005 Caenepeel and Femić introduced the Brauer group of Azumaya corings and proved that it is isomorphic to the mentioned middle term cohomology group. This resolves the deficiency in the cohomological interpretation of the Brauer group of a commutative ring. In 2008 the same authors constructed a version of the infinite exact sequence for a commutative bialgebroid and we interpreted the middle terms on the first three levels of the sequence. If $R \to S$ is a commutative ring extension, then $S \otimes_R S$ is a commutative bialgebroid and the new sequence generalizes the previous one.

In the present work we introduce a cohomology for a braided finite tensor category and construct an infinite exact sequence of the respective cohomology groups. We investigate how much of the previous results can be recovered in this setting. This is a work in progress.

Lie theory beyond Lie groups

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In the last 25 years a new type of Lie theory, that goes beyond the usual Lie group methods, has emerged. The basic global objects in this theory are called *Lie groupoids* and their infinitesimal counterparts are called *Lie algebroids*. In this lecture I will survey some of the highlights of Lie groupoid/algebroid theory as well as some of its potential applications.

Painleve IV coherent states

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Joint work with: David Bermudez, Alonso Contreras-Astorga

Schrodinger Hamiltonians which have third order differential ladder operators give place to solutions to the Painlevé IV equation. Some of them appear from applying SUSY QM to the harmonic oscillator Hamiltonian. Once they have been generated through this technique, we build their coherent states as eigenstates of the annihilation operator, then as displaced versions of the extremal states, both involving the third-order ladder operators, and finally as displaced extremal states but using a pair of linearized ladder operators. To each SUSY generated Hamiltonian corresponds two families of coherent states: one inside the infinite subspace associated with the oscillator spectrum and another one in the finite subspace generated by the states created through SUSY QM.

On the nonlinear differential equations

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Certain special functions are solutions of nonlinear differential equations (for instance, the Painleve equations, which are in turn connected with the integrable systems.) The solutions of the Painleve equation have movable poles. If one considers general nonlinear differential equations, then solutions might have complicated singularities structure. The talk will be based on [G. Filipuk, R. Halburd, Movable Singularities of Equations of Lienard Type, Computational Methods and Function Theory Volume 9(2009), No. 2, 551â563].

Some examples of covariant integral quantization

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We describe and apply integral quantization, a procedure based on operator-valued measures and the resolution of the identity. We explore its covariance properties in the context of group representation theory. Three applications based on group representations are carried out. The first one concerns the covariant integral quantization based on the spin one-half irreducible representation of SU(2). In this case we show that the quantization of both the quadratic Hopf variables and the Euler angles reproduces the SU(2) Lie algebra. The second example revisits integral quantization based on the Weyl-Heisenberg group. By completing previous results, we show the universality of the canonical commutation relation in such quantizations. In the last example we revisit and enrich the integral quantization based on the affine group of the real line and we give a short account of a relevant application.

Enumeration of Vacua in String Theory via Algebraic Combinatorics

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How to count vacua is one of the most interesting questions that can arise in the study of the string landscape. Resolutions to this question will enhance our ability to classify the crowded string landscape and pinpoint the desired vacuum that represents the gauge group of the world, namely, the standard model. In this talk, we introduce the progress that we made on this question by translating it into an equivalent problem in algebraic combinatorics that has not been posed before in the literature. We do so in the context that involves the moduli space of M-theory compactifications on singular manifolds with G_2 holonomy; it takes into consideration dualities between gauge theories with different gauge groups but equal numbers of $U(1)$ factors. We show that counting these dual vacua is equivalent to enumerating conjugacy classes of elements of finite order inside Lie groups. We pose and solve these enumeration problems. We are also able to point out that symmetry breaking patterns by Wilson lines and Higgs fields are different in certain cases, unlike the conventional expectation that they are the same.

Critical phenomena in small world networks

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Joint work with: Progulova T.B.

We study the problem of broken symmetry in small world networks. We describe the symmetry of the regular subgraph small world system as discrete subgroup of the Galilean group. The integer basis of the irreducible representation under consideration determines the type of the expansion of the free energy functional in power of the order parameter. Taking into account the presence of the random connections in the small world system the free energy functional can be written as

$F[\eta] = F_0[\eta] + F_I[\eta]$,

where

$F_0[\eta] = \iint drdr'dtdt' \left[\frac{\partial\eta(r,t)}{\partial t} k(r,t,r',t') \frac{\partial\eta(r',t')}{\partial t'} + \frac{\partial\eta(r,t)}{\partial r} k_0(r,t,r',t') \frac{\partial\eta(r'.t')}{\partial r'} \right]$

and

$F_I[\eta] = \iint drdr'dtdt' V\,(\eta(r,t), \eta(r',t'))$.

Here we took into account the presence of non-local connections and $\eta(r,t)$ is an order parameter. The equation of motion for the order parameter is derived from the stationary principle, namely, the Gateaux derivative $\delta F[\eta, h] = 0$ for any h [1]. This condition leads to an integro-differential equation for the order parameter. We define the "kernel" in this equation using the classification of generalized entropy and the maximum entropy principle [2]. We prove that a small world fractal structure is characterized by the Shafee entropy. In the case of the uniform distribution of fractal dimensions of substructures in a multifractal structure we obtain the Tsallis entropy. We derive equations to determine the kind of generalized entropies for the other small world structures. In the case of the kernel having a power law form, we obtain a fractional nonlinear differential equation. We study the symmetry properties of the fractional nonlinear differential equations and analyze a spatial distribution of the order parameter in detail.

We take into consideration the influence of the small world property on the phase transition and obtain a nonlinear dispersion law. Within the framework of the renormalization group method, this property taken into account makes it possible to obtain a critical indexes dependence on the parameter of the system complexity.

Based on the fMRI data, we construct brain functional networks and try to describe the properties of these networks by using our results.

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Twisted symmetries of differential equations

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Symmetry methods are a powerful method to obtain general reduction or special solutions for nonlinear differential equations. In recent years, several kind of "twisted symmetries", based on a modification of the prolongation operation, have been considered (starting with the "λ-symmetries" of Muriel and Romero), and proved to be as effective as standard symmetries for the tasks mentioned above. We provide a unified description of these twisted symmetries, and discuss to what extent they may be further generalized.

Rotating Taub-bolt instanton revisited

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Joint work with: Gérard Clément

We study the solutions of the Euclidean signature vacuum Einstein equations which are obtained locally by analytic continuation of the Lorentzian Kerr-Taub-Nut metrics. These contain three independent parameters: the mass, the NUT parameter and the rotation parameter. To be interpreted as gravitational instantons they must be further constrained to avoid conical and Misner string singularities, which leads to the two-parametric family of instanton solutions. We critically revisit earlier suggestions for the constraint showing that successful elimination of singularities in the Euclidean solution requires an additional singular coordinate transformation of the initial Kerr-Taub-Nut metric to be done. Contrary to previous results, our constraint equation can be solved analytically and describes three families of the globally regular rotating Taub-bolt instantons. Additional restrictions on the parameters following from the absence of curvature singularities are derived.

Routh reduction, compatible transformations and stages

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Joint work with: Frans Cantrijn, Bavo Langerock

In this talk, we introduce a geometric framework for Routh reduction of Lagrangian systems with symmetry. Our approach is based on a class of mappings that we call *compatible transformations*, and which allow for a systematic treatment of this reduction technique. In particular, we discuss some aspects of Routh reduction by stages.

Group theoretical methods in liquid crystal dynamics

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In this talk I will report a recent advance made in the theory of liquid crystals with the help of group theoretical methods and symmetries.

There are two competing descriptions of nematic liquid crystal dynamics: the Ericksen-Leslie director theory and the Eringen micropolar approach. Up to this day, these two descriptions have remained distinct in spite of several attempts to show that the micropolar theory comprises the director theory. In this talk we will show that this is the case by using Lie group symmetry reduction techniques and introducing a new system that is equivalent to the Ericksen-Leslie equations and includes disclination dynamics. More precisely, we will show how these systems can be seen as symmetry reduced Euler-Lagrange equations on a semidirect product involving the diffeomorphism group and the gauge group of the theory, and how this geometric approach can be used to prove that the micropolar theory of liquid crystal comprises the well-known Ericksen-Leslie theory, by applying two symmetry reduction processes. The resulting equations of motion are verified to be completely equivalent, although one of the two offers the possibility of accounting for orientational defects. Our results also apply to other models such as the ordered micropolar theory of Lhuiller and Rey.

Covariant integral quantizations

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Joint work with: Hervé Bergeron, Université Paris-Sud

Integral quantization is a procedure based on operator-valued measure and resolution of the identity. We insist on covariance properties in the important case where group representation theory is involved. We also insist on the inherent probabilistic aspects of this classical-quantum map. The approach includes and generalizes coherent state quantization. Two applications based on group representation are carried out. The first one concerns the Weyl-Heisenberg group and the euclidean plane viewed as the corresponding phase space. We show that a world of quantizations exist, which yield the canonical commutation rule and the usual quantum spectrum of the harmonic oscillator. The second one concerns the affine group of the real line and gives rise to an interesting regularization of the dilation origin in the half-plane viewed as the corresponding phase space. An interesting application to quantum cosmology will be presented.

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A Laplace-Dunkl equation on the 2-sphere and the Bannai-Ito algebra

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Joint work with: Luc Vinet and Alexei Zhedanov

In this talk, I shall explain how the analysis of the $\mathbb{Z}_2^3$ Laplace-Dunkl equation on the 2-sphere can be cast in the framework of the Racah problem for the $sl_{-1}(2)$ algebra. I will show how the Bannai-Ito polynomials and algebra arise in this context.

On description of the orbit space of unitary actions for mixed quantum states

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Joint work with: Arsen Khvedelidze and Yuri Palii

The space of mixed states, $\mathfrak{P}_+$, of n-dimensional binary quantum system is locus in quo for two unitary groups action: the group $\mathrm{U}(n)$ and the tensor product group $\mathrm{U}(n_1) \otimes \mathrm{U}(n_2)$, where n_1, n_2 stand from dimensions of subsystems, $n = n_1 n_2$. Both groups act on a state $\varrho \in \mathfrak{P}_+$ in adjoint manner $(\mathrm{Ad}\, g)\varrho = g\,\varrho\, g^{-1}$. As a result of this action one can consider two equivalent classes of ϱ; the *global* $\mathrm{U}(n)-$orbit and the *local* $\mathrm{U}(n_1) \otimes \mathrm{U}(n_2)-$orbit. The collection of all $\mathrm{U}(n)$-orbits, together with the quotient topology and differentiable structure defines the "global orbit space", $\mathfrak{P}_+ \,|\mathrm{U}(n)$, while the orbit space $\mathfrak{P}_+ \,|\mathrm{U}(n_1) \otimes \mathrm{U}(n_2)$ represents the "local orbit space", or the so-called *entanglement space* $\mathcal{E}_{n_1 \times n_2}$. The latter space is proscenium for manifestations of the intriguing effects occurring in quantum information processing and communications.

Both orbit spaces admit representations in terms of the elements of integrity basis for the corresponding ring of group-invariant polynomials. This can be done implementing the Processi and Schwarz method, introduced in the 80th of last century for description of the orbit space of a compact Lie group action on a linear space. According to the Processi and Schwarz the orbit space is identified with the semi-algebraic variety, defined by the syzygy ideal for the integrity basis and the semi-positivity condition of a special, so-called "gradient matrix", $\mathrm{Grad}(z) \geqslant 0$, constructing from the integrity basis elements.

In the present talk we address the question of application of this generic approach to the construction of $\mathfrak{P}_+ \,|\mathrm{U}(n)$ and $\mathfrak{P}_+ \,|\mathrm{U}(n_1) \otimes \mathrm{U}(n_2)$. Namely, we study whether the semi-positivity of $\mathrm{Grad}-$ matrix introduces a new conditions on the elements of the integrity basis for the corresponding ring $\mathbb{R}[\mathfrak{P}_+]^{\mathrm{G}}$.

Generalized hydrodynamical variables for quantum mechanics, and the group of nonlinear gauge transformations

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The group of nonlinear gauge transformations, introduced in 1996 by H.-D. Doebner and the author, acts on the Hilbert manifold of quantum-mechanical wave functions. Such a nonlinear gauge transformation alters the phases of the wave functions, changes the time-evolution equation, and modifies the expressions for observables, in such a way as to leave all of the outcomes of physical measurements (as described by the appropriate measurement theory) invariant. The nonlinearities entering the Schrödinger equation involve functionals that are suggestive of Madelung's "hydrodynamical" formulation. More generally, a widened class of nonlinear time-evolution equations based on such functionals unifies a large set of previously-proposed ways to introduce nonlinearity into quantum mechanics. The resulting family of nonlinear derivative Schrödinger equations can be re-expressed in terms of hydrodynamical variables in a manifestly gauge-invariant way. Among the resulting new features, the quantum potential term is governed by two independent coefficients instead of one; there is a dissipative term that moves us from Euler to Navier-Stokes hydrodynamics; and external forces can be exerted by two additional vector fields. An explicit frictional term originates with Kostin's version of nonlinear quantum mechanics. This description provides particular reasons for renewed interest in the role of nodal sets (the zeroes of wave functions) in both linear and nonlinear quantum mechanics, which I shall discuss.

Backlund Transformation for mKdv Hierarchy

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A systematic construction of Integrable hierarchies is proposed in terms of an Affine Kac-Moody algebra. In particular we shall discuss the mKdV hierarchy and its higher nonlinear solitonic equations. We explicitly construct the Backlund transformation for the mKdV equation as well as for the first few higher members of the hierarchy.

The exact Foldy-Wouthuysen transformation for a Dirac theory with the complete set of CPT/Lorentz violating terms

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From the point of view of a better understanding and interpretation of physical properties of Dirac and Pauli's equations, it is worth considering nonrelativistic approximation of the theory with corresponding to CPT-Lorentz breaking terms. The FW method is a direct calculation that allows to extract physical information of the Dirac action. One can now think about the possibility of studing the other different terms that break CPT-Lorentz. The main point would be the search form terms that have the same behavior in Dirac action as the torsion field. Knowing how this term would appear on the respective Hamiltonian, the next step would be just to perform the physical analysis. But, the algebra involved in the derivation of the explicitly form of the Hamiltonian that may give direct physical result is not straightforward.

There is an algorithm that shows how to construct this general Hamiltonan. The Hamiltonian will be the one that accepts the EFWT and therefore allows one to extract physical parameters. On the same paper, the authors presented the complete 80 new cases of CPT-Lorentz violating terms in the Dirac equation. The focus of the present work is precisely the development of EFWT, considering the whole situation presented in the last related work as well as the calculation of equations of motion.

We also explore the possibility of combining such equations in order to obtain an expression to describe the particle's dynamic. Although FWT provides, in general, more detailed information about the nonrelativistic approximation, there is a considerable advantage in the construction of the EFWT because in this case, we do not run the risk of missing some important terms for the spinor field in the weak gravitational field case. However the possibility of performing exact transformation depends on the existence of the so called involution operator on the external classical fields and from the mathematical point of view EFWT is more complex.

Taking into account the context explained above, we consider in the present work the complete 80 cases of CPT-Lorentz violating terms in the Dirac equation. Our aim here is to develop the EFWT for the most complete action involving the CPT-Lorentz violation. The case of torsion is mainly considered in other papers. One can use the calculation procedure adopted here as a guidance to perform EWFT in the situation where a large quantity of odd terms are present, once that in this kind of situation the EFWT is not quite trivial.

In this work we present a brief summarize about EWFT. Coming up we consider the 80 cases mentioned above of CPT-Lorentz violating terms in the Dirac equation and perform EWFT. Finally we obtain the bound that gives the indication of the most coerent possible experiment to measure the physical properties of the Dirac equation.

Higher Spin Algebras in AdS5 and AdS7

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Joint work with: Prof. Murat Günaydin

Eastwood (2002) has defined the AdS_d/CFT_{d-1} higher spin algebra $hs(d,2)$ as the universal enveloping algebra $\mathcal{U}(\mathfrak{so}(d-1,2))$ of the Lie algebra $\mathfrak{so}(d-1,2)$ quotiented by an ideal $\mathcal{J}(\mathfrak{so}(d-1,2))$ that annihilates the minimal unitary representation (minrep) of $\mathfrak{so}(d-1,2)$. We reformulate the minreps of $\mathfrak{so}(4,2)$ and $\mathfrak{so}(6,2)$ Lie algebras, as obtained by Fernando and Günaydin using quasiconformal methods, in terms of bilinears of deformed twistors that transform nonlinearly under the Lorentz groups $\mathfrak{so}(3,1)$ and $\mathfrak{so}(5,1)$ and explicitly show that their Joseph ideals vanish identically as operators. Consequently their enveloping algebras yield the higher spin algebras $hs(4,2)$ and $hs(6,2)$ directly within the quasiconformal approach. Furthermore, the quasiconformal approach provides a unitary realization of the corresponding higher spin algebras. Corresponding deformed twistors can be thought of as generalizations of Wigner's deformed oscillators with operator valued "deformation parameters".

We also show that there exists a continuous one parameter family of deformations of $hs(4,2)$ labeled by the helicity $\zeta/2$ of corresponding massless conformal fields in four dimensions and a discrete infinite family of deformations of $hs(6,2)$ labeled by the spin t of an $\mathfrak{su}(2)$ subgroup of the little group $SO(4)$ of massless particles in six dimensions. We argue that the holographic duals of the corresponding interacting higher spin theories in AdS_5 and AdS_7 must be interacting (and possibly integrable) conformal field theories in four and six dimensions respectively.

The results discussed in this talk are based on the joint work with Murat Günaydin which appeared in arXiv:1312.2907 and arXiv:1401.6930.

Low and Higher Energy Limits of Electroweak Model as Contraction of its Gauge Group

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The Electroweak Model is a gauge theory based on the group $SU(2) \times U(1)$, acting in the boson, lepton and quark sectors. Vectors of the fundamental representaion space C_2 have different physical interpretations in different sectors. The contracted group $SU(2;j)$ and its fundamental representation space $C_2(j)$ are obtained by the **consistent rescaling** of $SU(2)$ and C_2

$$z'(j) = \left(\begin{array}{c} jz_1' \\ z_2' \end{array} \right) = \left(\begin{array}{cc} \alpha & j\beta \\ -j\bar{\beta} & \bar{\alpha} \end{array} \right) \left(\begin{array}{c} jz_1 \\ z_2 \end{array} \right) = u(j)z(j), \tag{1}$$

when the contraction parameter tends to zero $j \to 0$. The space $C_2(j)$ is obtained from C_2 by the substitution $z_1 \to jz_1$, which corresponds to the similar transformation of components of the lepton $L_l = \left(\begin{array}{c} \nu_l \\ e_l \end{array} \right)$ and the quark $Q_l = \left(\begin{array}{c} u_l \\ d_l \end{array} \right)$ fields, namely: $\nu_l \to j\nu_l,\ e_l \to e_l,\ u_l \to ju_l,\ d_l \to d_l$. The rescaling (1) induces the substitution of the boson fields $W_\mu^{\pm} \to jW_\mu^{\pm},\ Z_\mu \to Z_\mu,\ A_\mu \to A_\mu$. The right lepton and quark fields are $SU(2)$-singlets, i.e. scalars, and therefore are not transformed. In this scheme the contraction parameter is connected with the energy s in center-of-mass system and is evaluated through the fundametal parameters of the Electroweak Model $j^2(s) = g\sqrt{s}/m_W$, where m_W is W-boson mass and g is constant [1]. So contraction $j \to 0$ corresponds to zero energy limit of the Electroweak Model.

In the limit $j \to 0$ the first components of the lepton and quark doublets become infinitely small in comparison with their second components. On the contrary, when energy increases the first components of the doublets become greater then their second ones. The infinite energy limit corresponds to the **new consistent rescaling** of $SU(2)$ and C_2

$$z'(\epsilon) = \left(\begin{array}{c} z_1' \\ \epsilon z_2' \end{array} \right) = \left(\begin{array}{cc} \alpha & \epsilon\beta \\ -\epsilon\bar{\beta} & \bar{\alpha} \end{array} \right) \left(\begin{array}{c} z_1 \\ \epsilon z_2 \end{array} \right) = u(\epsilon)z(\epsilon), \tag{2}$$

In the second contraction scheme (2) all gauge bosons are transformed according to the same rules with the natural substitution of j by ϵ. But the lepton and quark fields are transformed as follows: $e_l \to \epsilon e_l,\ d_l \to \epsilon d_l,\ \nu_l \to \nu_l,\ u_l \to u_l$. The next reason for inequality of the first and second doublet components is the special mechanism of spontaneous symmetry breaking, which is used to generate mass of vector bosons and other elementary particles of the model. In this mechanism one of Lagrangian ground states $\phi^{vac} = \left(\begin{array}{c} 0 \\ v \end{array} \right)$ is taken as vacuum of the model and then small field excitations $v + \chi(x)$ with respect to this vacuum are regarded. So Higgs boson field χ and the constant v are multiplied by ϵ. As far as masses of all particlies are proportionate to v we obtain the following transformation rule for contraction (2) $\chi \to \epsilon\chi, \quad v \to \epsilon v, \quad m_p \to \epsilon m_p$, where $p = \chi, W, Z, e, u, d$.

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Crossed products of C*-algebras for singular actions

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We consider group actions $\alpha : G \to \mathrm{Aut}(\mathcal{A})$ of topological groups G on C^*-algebras $\mathcal{A}$ of the type which occur in many quantum physics models. These are singular actions in the sense that they need not be strongly continuous, or the group need not be locally compact. We develop a "crossed product host" in analogy to the usual crossed product for strongly continuous actions of locally compact groups, in the sense that its representation theory is in a natural bijection with the covariant representation theory of the action $\alpha : G \to \mathrm{Aut}(\mathcal{A})$. We have a uniqueness theorem for crossed product hosts, as well as existence conditions. We have a number of examples where a crossed product host exists, but the usual crossed product does not, and if time permits, will consider a few.

Noncommutative Instantons and Reciprocity

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Joint work with: Toshio Nakatsu (Setsunan University)

In this talk, we discuss U(N) instantons in noncommutative (NC) spaces. Noncommutative space is a space which coordinate ring is noncommutative. Let x^μ be the spacial coordinates. The noncommutativity is expressed by the following commutation relations: $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. where $\theta^{\mu\nu}$ is a real antisymmetric tensor and called noncommutative parameters. When $\theta^{\mu\nu}$ vanishes identically, the coordinate ring is commutative and the underlying space reduces to a commutative one. The commutation relations, like the canonical commutation relations in quantum mechanics, lead to "space-space uncertainty relation." Singularities in commutative space could resolve in noncommutative space thereby. This is one of the prominent features of field theories on noncommutative space and yields various new physical objects such as U(1) instantons. There are two formalism to describe noncommutative gauge theories: the star-product formalism and the operator formalism.

Anti-self-dual (ASD) Yang-Mills equation and the solutions have been studied from the several viewpoints of mathematical physics, particularly, integrable systems, geometry and field theories. Instantons are finite-action solutions of the ASD Yang-Mills equa- tion and become exact solutions of classical Yang-Mills theories. They can reveal non-perturbative aspects of the quantum theories. Actually, the path-integrations, formulating the quantum theories, could reduce to finite-dimensional integrations over the instanton moduli spaces. The Atiyah-Drinfeld-Hitchin-Manin (ADHM) construction is a powerful method to obtain the instantons. Furthermore, via the construction, the instanton moduli space is mapped to the set of quadruple matrices which are solutions of the ADHM equation and called the ADHM data. The aforementioned integration, being thereby an integration over the matrices, becomes tractable. To evaluate the integration, the use of noncommutative instantons is relevant so that a localization formula can be applied to the integration. In the procedures, various formulas and relations of the ADHM construction are required. Hence it is worthwhile to elucidate the one-to-one correspondence (reciprocity) between moduli spaces of the noncommutative instantons and the ADHM data and to present all the ingredients in the construction explicitly.

We prove the noncommutative reciprocity in the ADHM construction in both the star-product formalism and the operator formalism. We also discuss the construction of exact solutions and an origin of the instanton number. Partial results are seen in arXiv:1311.5227 [hep-th] and our forthcoming papers.

2D Toda tau functions as combinatorial generating functions

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Joint work with: Mathieu Guay-Paquet

Two methods of constructing 2D Toda τ-functions that are generating functions for certain geometrical invariants of a combinatorial nature are related. The first involves generation of paths in the Cayley graph of the symmetric group S_n by multiplication of the basis elements $C_\lambda \in \mathbf{C}[S_n] \otimes \mathbf{C}[S_n]$ of the group algebra of the symmetric group $\mathbf{C}[S_n]$ consisting of sums over each conjugacy class by elements of an abelian group of central elements. Extending the characteristic map to the tensor product $\mathbf{C}[S_n] \otimes \mathbf{C}[S_n]$ leads to double expansions over products of monomial symmetric functions, in which the coefficients count the number of such paths. The second method is the standard construction of hypergeometric τ-functions as vacuum state matrix elements of products of vertex operators in a fermionic Fock space with elements of the abelian group of *convolution symmetries*. A homomorphism between these two group actions is derived and shown to be intertwined by the characteristic map composed with fermionization. Applications include Okounkov's generating functions for double Hurwitz numbers, which count branched coverings of the Riemann sphere with nonminimal branching at two points, and various analogous combinatorial counting functions.

The quantum formulation derived from assumptions of epistemic processes.

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An epistemic process is any process under which one obtains knowledge about a physical system. A conceptual variable is any variable related to the system, defined by an actor or a set of communicating actors. An e-variable θ is a conceptual variable associated with an epistemic process: Before the process one has no knowledge about θ; after the process one has some knowledge. A conceptual variable ϕ is called inaccessible if there is no epistemic process by which one can get accurate knowledge about it; an e-variable θ is called accessible if there exists a process by which one can get accurate knowledge about it. This is related to the complementarity concept of Bohr. Note that ϕ is not a hidden variable in a physical sense; it is just an abstract variable in a mathematical space. I am using Greek letters to indicate an extension of the statistical parameter-concept.

Let in general ϕ be an inaccessible conceptual variable taking values in some topological space Φ, and let $\theta^a = \theta^a(\phi)$ be accessible functions for a belonging to some index set A. All functions on Φ are assumed to be Borel-measurable. To begin with I assume that the functions θ^a are maximal and also that there is an isomorphism between them. Later this is generalized. Weak assumptions are made on groups G^a and G of transformation on Φ. Any example with spin or angular momentum for a particle or for a set of particles satisfies these assumptions.

Concentrate on the case where each λ^a takes a discrete set of values. In Helland (2014) the following is proved: The epistemic states defined by 1) A question "What is the value of θ^a?" and 2) The ideal answer "$\theta^a = u_k$", can be put in one-to-one correspondence with formal pure quantum states in a concrete Hilbert space H, a subspace of $L^2(\Phi, \rho)$, where ρ is the invariant measure on Φ induced by G. The operator A^a corresponding to the e-variable λ^a can then be defined as the operator with eigenvalues u_k and corresponding eigenspaces constructed by projections on the quantum states above.

Continuing in the same setting, Born's formula and the Schrödinger equation are derived from natural assumptions. The controversies related to Bell's inequality are resolved using a generalization the conditionality principle from statistics. Several of the so-called paradoxes in quantum mechanics are shown to have simple solutions in the epistemic setting. I will claim that the foundation defined from the symmetrical epistemic setting above is more intuitive than the ordinary formal textbook foundation of quantum theory.

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Symbolic Computation of Conservation Laws of Nonlinear Partial Differential Equations

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A method will be presented for the symbolic computation of conservation laws of nonlinear partial differential equations (PDEs) involving multiple space variables and time.

Using the scaling symmetries of the PDE, the conserved densities are constructed as linear combinations of scaling homogeneous terms with undetermined coefficients. The variational derivative is used to compute the undetermined coefficients. The homotopy operator is used to invert the divergence operator, leading to the analytic expression of the flux vector.

The method is algorithmic and has been implemented in the syntax of the computer algebra system Mathematica. The software is being used to compute conservation laws of nonlinear PDEs occurring in the applied sciences and engineering.

The software package will be demonstrated for PDEs that model shallow water waves, ion-acoustic waves in plasmas, sound waves in nonlinear media, and transonic gas flow. The featured equations include the Korteweg-de Vries, Kadomtsev-Petviashvili, Zakharov-Kuznetsov, and Khoklov-Zabolotskaya equations.

Tensor Powers of Galois Corings

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Joint work with: Lars Kadison, Marcin Szamotulski

We show that the subgroup depth of a finite subgroup pair $H < G$ over a field is greater than the depth of the crossed product algebra extension $AH < AG$ with respect to a 2-cocycle. The most convenient theoretical underpinning to do so is provided by the entwining structure of a right H-comodule algebra A and a right H-module coalgebra C over a Hopf algebra H. Then $A \otimes C$ is an A-coring, which is Galois in certain cases when C is the quotient module Q of a Hopf subalgebra $R < H$. We show that this is the case for the crossed product algebra extension above, in which case the depth of this Galois coring is less than the depth of H in G. We note that h-depth of a subgroup pair and its coreless quotient pair are equal.

An exactly solvable deformation of the Coulomb problem associated with the Taub-NUT metric

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Joint work with: Angel Ballesteros, Alberto Enciso, Orlando Ragnisco and Danilo Riglioni

We consider the N-dimensional maximally superintegrable classical Hamiltonian system given by [1]

$$\mathcal{H} = \frac{|\mathbf{q}|\mathbf{p}^2}{2(\eta + |\mathbf{q}|)} - \frac{k}{\eta + |\mathbf{q}|},$$

which can be regarded as a smooth deformation of the Coulomb potential with coupling constant k on the Taub-NUT space [2]. The vanishment of the deformation parameter η leads to the Coulomb potential on the Euclidean space.

We face the quantization of $\mathcal{H}$ by requiring to preserve the maximal superintegrability property, thus providing a new exactly solvable deformation of the quantum Coulomb problem [3]. In particular, we prove that this strong symmetry condition is fulfilled by the so-called conformal Laplace–Beltrami quantization prescription, where the conformal Laplacian contains the usual Laplace–Beltrami operator on the Taub–NUT manifold plus a term proportional to its scalar curvature. In this way, the eigenvalue problem for the quantum counterpart of $\mathcal{H}$ is rigorously and fully solved for positive values of η and k. It is found that its discrete spectrum is just a smooth deformation (in terms of the Taub-NUT parameter η) of the N-dimensional Coulomb spectrum. Moreover, it turns out that the maximal degeneracy of the flat Coulomb system is preserved under deformation.

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The orthogonal planes split of quaternions and its relation to quaternion geometry of rotations

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Recently the general orthogonal planes split with respect to any two pure unit quaternions $f, g \in \mathbb{H}$, $f^2 = g^2 = -1$, including the case $f = g$, has proved extremely useful for the construction and geometric interpretation of general classes of double-kernel quaternion Fourier transformations (QFT) [E.Hitzer, S.J. Sangwine, The orthogonal 2D planes split of quaternions and steerable quaternion Fourier Transforms, in E. Hitzer, S.J. Sangwine (eds.), "Quaternion and Clifford Fourier Transforms and Wavelets", TIM **27**, Birkhauser, Basel, 2013, 15–39.]. Applications include color image processing, where the orthogonal planes split with $f = g =$ the grayline, naturally splits a pure quaternionic three-dimensional color signal into luminance and chrominance components. Yet it is found independently in the quaternion geometry of rotations [L. Meister, H. Schaeben, A concise quaternon geometry of rotations, MMAS 2005; **28**: 101–126], that the pure quaternion units f, g and the analysis planes, which they define, play a key role in the spherical geometry of rotations, and the geometrical interpretation of integrals related to the spherical Radon transform of probability density functions of unit quaternions, as relevant for texture analysis in crystallography. In our contribution we further investigate these connections.

Generalization of Weyl group orbit functions using matrix immanants

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Joint work with: Agnieszka Tereszkiewicz

Matrix immanants are a generalization of the determinant and permanent of a matrix. In this work we use immanants to extend families of symmetric and antisymmetric orbit functions related to Weyl groups of simple Lie algebras A_n. We describe properties of these functions and their relations to Weyl group orbit functions.

Plane-parallel waves as duals of the flat background

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Joint work with: I. Petr

Several exactly solvable string sigma models are found using non-Abelian T-duality. After dualizing the flat background with respect to four-dimensional subgroups of Poincare algebra, we identify most of T-dual models as conformal sigma models in plane-parallel wave backgrounds, some of them having torsion. We give their standard form in Brinkmann coordinates and find exact solutions to classical string equations of motion.

On some differential transformations of hypergeometric equations

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Joint work with: André Ronveaux

Many algebraic transformations of the hypergeometric equation $\sigma(x)z''(x) + \tau(x)z'(x) + lz(x) = 0$ with polynomial coefficients of degree $2, 1, 0$, respectively, are well known. Some of them act on $x = x(t)$, in order to recover the Heun equation, similar to the equation satisfied by the hypergeometric function ${}_2F_1$, but with polynomial coefficients of degree $3, 2, 1$, investigated by K. Kuiken and R. S. Maier. The transformations by the function y(x) =A(x)z(x), also very popular in mathematics and physics, are used to get, for instance, Schroedinger equations with appropriate potentials, but also Heun and confluent Heun equations. This work addresses a generalization of Kimura's approach proposed in 1971, based on differential transformations of the hypergeometric equations involving y(x)=A(x)z(x) + B(x) z'(x). Appropriate choices of A(x) and B(x) allow to retrieve the Heun equations as well as equations for some exceptional polynomials.

Generalized Trigonometric Transforms of affine Weyl Groups

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Joint work with: Tomasz Czyżycki

We discuss the properties of transforms of special functions related to the root systems of simple Lie algebras and the corresponding affine Weyl groups. The four types of special functions, formed by certain finite sums of exponential functions, are generalizations of one-dimensional sine and cosine functions. We consider certain generalization of four types of common discrete cosine (DCT) and sine transforms (DST) to these special functions. We show that such a generalization is possible for two infinite series B_n and C_n. A special case of the algebra C_2, where twelve new transforms are obtained, is exemplified. Related discrete Fourier analysis on lattices is formulated.

Entanglement created with generalized squeezed coherent states through a beam splitter

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Joint work with: A. Hertz and H. Eleuch

We analyse the entanglement created by generalized squeezed coherent states. Such quantum entanglement is generated using a beam splitter and measured by the linear entropy. Generalized squeezed states are constructed as eigenstates of general ladder operators associated with deformed Heisenberg algebras. We show the effects of the coherence and squeezing parameters variation on the measure of entanglement. We show that there is fundamental differences between finite and infinite superpositions of Fock states that build the generalized squeezed coherent states.

Positive representations and higher quantum Teichmüller Theory

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We review the notion of positive representations of split real quantum groups $\mathcal{U}_{q\tilde{q}}(\mathfrak{g}_{\mathbb{R}})$ introduced in a joint work with Igor Frenkel, and describe the tensor product decomposition when restricted to the Borel part. This generalized the essential step of the construction of the quantum Teichmüller theory from the representation of the quantum plane studied by Frenke-Kim, and provide a candidate for the quantum higher Teichmüller theory.

Characteristic identities for Lie (super)algebras

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Joint work with: Mark Gould and Jason Werry

This talk will present an overview of characteristic identities for Lie algebras and superalgebras. We outline methods that employ these characteristic identities to deduce matrix elements of finite dimensional representations. To demonstrate the theory, we look at examples of both Lie algebras and Lie superalgebras.

The constitutive tensor of linear elasticity: its decompositions, Cauchy relations, null Lagrangians, and wave propagation

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Joint work with: F.-W. Hehl (Cologne)

In linear anisotropic elasticity, the elastic properties of a medium are described by the fourth rank elasticity tensor C. The decomposition of C into a partially symmetric tensor M and a partially antisymmetric tensors N is often used in the literature. An alternative, less well-known decomposition, into the completely symmetric part S of C plus the reminder A, turns out to be irreducible under the 3-dimensional general linear group. We show that the SA-decomposition is unique, irreducible, and preserves the symmetries of the elasticity tensor. The MN-decomposition fails to have these desirable properties and is such inferior from a physical point of view. Various applications of the SA-decomposition are discussed: the Cauchy relations (vanishing of A), the non-existence of elastic null Lagrangians, the decomposition of the elastic energy and of the acoustic wave propagation. The acoustic or Christoffel tensor is split in a Cauchy and a non-Cauchy part. The Cauchy part governs the longitudinal wave propagation. We provide explicit examples of the effectiveness of the SA-decomposition. A complete class of anisotropic media is proposed that allows pure polarizations in arbitrary directions, similarly as in an isotropic medium.

su(2) Krawtchouk oscillator model under the CP deformed symmetry

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Joint work with: A.M. Jafarova, J. Van der Jeugt

Krawtchouk polynomials $K_n(x; p, j)$ are defined through the ${}_2F_1$ type hypergeometric functions, and there is a well-known finite orthogonality relation for the them with respect to the discrete measure. Number of applications of them already exists to construct $su(2)$ and $sl(2|1)$ finite-discrete quantum harmonic oscillator models. In case of $su(2)$ dynamical symmetry, the wavefunctions of the model are defined through the special case of Krawtchouk polynomials with a fixed parameter $p = 1/2$, although the wavefunctions of the supersymmetric model with $sl(2|1)$ dynamical symmetry are expressed through the Krawtchouk polynomials with parameter $0 < p < 1$. In this work, we present the oscillator model, which wavefunctions of even and odd states are also expressed by the Krawtchouk polynomials with fixed $p = 1/2$, $K_{j+m}(j + k; 1/2, 2j)$ and $K_{j+m-1}(j + k - 1; 1/2, 2(j - 1))$. Their dynamical symmetry is CP deformation of $su(2)$ Lie algebra:

$$
\begin{aligned}
&[J_+, J_-] = 2J_0\,(1 - CP)\,,\\
&[J_0, J_\pm] = \pm J_\pm,
\end{aligned}
$$

with an algebra generators, defined as

$$
\begin{aligned}
&C|j,m\rangle = 2j\ |j,m\rangle,\\
&P|j,m\rangle = (-1)^{j+m}\ |j,m\rangle,\\
&J_0|j,m\rangle = m\ |j,m\rangle,\\
&J_+|j,m\rangle = \begin{cases} \sqrt{(j-m)(j-m-1)}\ |j,m+1\rangle, & \text{if } j+m \text{ is even;}\\ -\sqrt{(j+m)(j+m+1)}\ |j,m+1\rangle, & \text{if } j+m \text{ is odd,} \end{cases}\\
&J_-|j,m\rangle = \begin{cases} -\sqrt{(j+m)(j+m-1)}\ |j,m-1\rangle, & \text{if } j+m \text{ is even;}\\ \sqrt{(j-m)(j-m+1)}\ |j,m-1\rangle, & \text{if } j+m \text{ is odd.} \end{cases}
\end{aligned}
$$

We found that finite spectrum of the position operator $\hat{x}$ of the proposed model is given by $\pm\sqrt{j^2 - k^2}$, $k = 0, 1, \ldots, j$. We also investigated different properties of this model.

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Dorfman connections and Courant algebroids

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Linear connections are useful for describing the tangent spaces of vector bundles, especially their Lie algebroid structure. The direct sum of the tangent space and the cotangent space of a manifold carries the structure of a "standard Courant algebroid", which naturally extends the Lie algebroid structure of the tangent space. In geometric mechanics, it is often useful to understand the standard Courant algebroid over a vector bundle (e.g. a phase space T^*Q). I will introduce the notion of "Dorfman connection" and explain how the standard Courant algebroid structure over a vector bundle is encoded by a certain class of Dorfman connections. If time permits, I will give more examples, showing that Dorfman connections are natural objects in the study of Courant algebroids and Dirac structures.

Clifford Analysis for fractional Dirac operators

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Joint work with: P. Cerejeiras, N. Vieira

Classic Clifford analysis consists in the establishment of a function theory for functions belonging to the kernel of the Dirac operator, so-called monogenic functions. While such functions can very well describe problems of a particle with internal SU(2)-symmetries, such as electrons, higher order symmetries are not included in this theory. Although many modifications (such as Yang-Mills theory) were suggested over the years they could not address the principal problem, the need for an n-fold factorization of the d'Alembert operator. In this talk we propose a factorization based on generalized Clifford algebras and the Riemann-Liouville derivative which allows to construct a Dirac operator with SU(n) symmetries. Furthermore, we will establish the groundwork for the corresponding function theory of this operator.

Clifford analysis in precanonical quantization approach in quantum field theory

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I review my recent work on precanonical quantization in field theory and quantum gravity and the problems in Clifford analysis it naturally leads to. The approach is based on quantization of Poisson-Gerstenhaber brackets of differential forms, which were found within the De Donder-Weyl (covariant) Hamiltonian formulation of field theories. Quantization of differential forms leads to the multiparametric hypercomplex generalizaton of quantum mechanics to field theory in which the Schrödinger equation for time evolution is replaced by the Dirac-like equation for the wave function of quantum fields:

$$i\hbar\varkappa\gamma^{\mu}\partial_{\mu}\Psi = \hat{H}\Psi,$$

where $\hat{H}$ is the De Donder-Weyl Hamiltonian operator, Ψ is a Clifford-valued wave function on the space of fields and space-time variables, and $\varkappa$ is a parameter of the dimension $length^{(1-n)}$ in n space-time dimensions, which originates from the representation of differential forms by Clifford algebra elements. I show that the standard canonical quantization in field theory (in the functional Schrödinger representation) is obtained from this approach in the limit of infinite $\varkappa$. I also show that application of this approach to general relativity leads to a multivariable hypercomplex generalization of the confluent hypergeometric eqiation, which plays the role of the Schrödinger equation for quantum gravity, where the curved space-time Dirac matrices are represented by differential operators acting on the wave function on the space of spin-connection coefficients.

Re-gauging, symmetries and degeneracies for Graph Hamiltonians

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Joint work with: Ralph Kaufmann and Sergei Khlebnikov

We show how to derive a quantum system from a graph. In applications this is the Harper Hamiltonian for a solid state system. We then go on to show that this procedure has natural re-gauging symmetries, which we analyze. One upshot is that through these symmetries, we get quantum enhanced symmetries of the graph itself. We apply this to special systems like the honeycomb lattice and the Gyroid, which can be thought of as a 3d generalization. The results are that all the degeneracies in the spectrum of these systems can be found by the symmetries just described.

Hopf algebras related to categorical structures and number theory

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Joint work with: A. Tonks and I. Galvez-Carillo

We show that is a certain categorical framework naturally leads to Hopf algebras. These Hopf algebras are of combinatorial nature and in special examples are isomorphic to those found be Goncharov for multi-zeta values and those of Connes and Kreimer in renormalization.

Z_3-splitting of quantum harmonic oscillator and ternary generalization of the Heisenberg algebra

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We investigate a ternary, Z_3-graded generalization of the Heisenberg algebra. It turns out that introducing a non-trivial cubic root of unity, $j = e^{\frac{2\pi i}{3}}$, one can define two types of creation operators instead of one, accompanying the usual annihilation operator. The two creation operators are non-hermitian, but they are mutually conjugate. Together, the three operators form a ternary algebra, and some of their cubic combinations generate the usual Heisenberg algebra.

A set of eigenstates in coordinate representation is constructed in terms of functions satisfying linear differential equation of third order.

Bethe-Salpeter equation for unstable states

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We will present the formalism for the description of unstable states in quantum field theory, based on the Bethe-Salpeter equation.

Okounkov's BC-type interpolation Macdonald polynomials and their $q = 1$ limit

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Okounkov [1] defined BC-type interpolation Macdonald polynomials, which generalize shifted Macdonald polynomials and shifted Jack polynomials. Among others, he derived a combinatorial formula for these polynomials, and he derived a binomial formula which gives Koornwinder polynomials $P_\lambda(x;q,t;a_1,a_2,a_3,a_4)$ (a 5-parameter extension of BC-type Macdonald polynomials) as a sum of products of two BC-type interpolation Macdonald polynomials, one factor depending on q^λ and one factor depending on x. From this formula the duality property of Koornwinder polynomials is clear. Rains [2] used this binomial formula as definition for the Koornwinder polynomials and he derived all their other properties from this formula. In later work he defined elliptic analogues in a similar way.

What seems to have remained unobserved in the literature is that a straightforward $q = 1$ limit of BC-type interpolation Macdonald polynomials leads to the definition of what we may call BC-type interpolation Jack polynomials. The corresponding limit of the binomial formula for Koornwinder polynomials gives BC-type Jacobi polynomials $P_(x;\tau;\alpha,\beta)$ as a sum of products of a BC-type interpolation Jack polynomials depending on λ and a Jack polynomial depending on x. This formula was already given by Macdonald [3, (9.15)], but his combinatorial formula [3, p.58] for the first factor in the sum of products is different from the combinatorial formula which follows as a limit of the combinatorial formula for the BC-type interpolation Macdonald polynomials. It is this last formula which specializes in the rank 2 case to a balanced ${}_4F_3(1)$ expression in Koornwinder & Sprinkhuizen [4, Cor. 6.6].

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Group Theory for Embedded Random Matrix Ensembles: Open Problems

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Embedded random matrix ensembles, introduced in 1970 and being explored in considerable detail since 1994, have started occupying an important position in quantum physics in the study of isolated finite quantum many-particle systems. They are proved to be rich in their content and wide in their scope. Significantly, the study of embedded random matrix ensembles is still developing and partly this is due to the fact that mathematical tractability of these ensembles is still a problem. One class of embedded ensembles, for m number of fermions or bosons occupying N number of single particle states, are those generated by random two-body interactions with $SU(r)$ symmetry and $U(N) \supset U(\Omega) \otimes SU(r)$ embedding where $N = r\Omega$. For fermion systems with $r = 1$, 2 and 4 and boson systems with $r = 1$, 2 and 3 have been analyzed using Wigner-Racah algebra of the embedding algebra. Other ensembles that have received attention are those with embedding defined by $U(N) \supset SO(3)$, $U(N) \supset U(N_1) \oplus U(N_2)$; $N = N_1 + N_2$ and parity symmetry. Besides giving a overview of these random matrix ensembles, we will discuss some of the important open problems in the subject and these will include: (i) deriving formulas for the fourth and higher order moments of the one and two-point functions in eigenvalues of embedded Gaussian Unitary ensembles for many-particle systems generated by two-body interactions [EGUE(2)s]; (ii) deriving asymptotic results for the moments of the one and two-point functions of EGUE(2)s and more importantly for the corresponding Gaussian orthogonal ensembles EGOE(2)s [and EGOE(1+2)s with a mean-field] for both fermionic and bosonic ensembles; (iii) constructing and analyzing embedded sympletic ensembles EGSEs with $Sp(N)$ symmetry in two-particle spaces; (iv) developing in detail the pairing algebras for boson systems with spin (1/2 and 1 $\hbar$) degree of freedom. Let us add that #(i) calls for new advances in the general $SU(N)$ Wigner-Racah algebra and similarly, #(ii) calls for development of suitable asymptotic methods for the Wigner-Racah algebra of $U(N) \supset U(\Omega) \otimes SU(r)$ and other embedding algebras. Finally, future in the subject of embedded random matrix ensembles in quantum physics is exciting with enormous scope for developing new group theoretical methods for their analysis and with the possibility of their applications in, besides atomic, nuclear and mesoscopic physics, newer areas of research such as quantum information science, Bose gasses, statistical mechanics of isolated finite quantum systems and quantum many-body chaos.

Products of random matrices

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Joint work with: Dries Stivigny

It is a relatively recent insight that eigenvalues or singular values of products of random matrices may have a determinantal structure. I will discuss this for singular values of products of Ginibre random matrices. A similar result for products of truncated unitary matrices relies on the evaluation of a certain integral over the unitary group, which I will present as an open problem.

Tetrahedron equations and quantum R matrices for q-oscillator representations

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Joint work with: Masato Okado

The intertwiner of the quantized coordinate ring $A_q(sl_3)$ is known to yield a solution to the tetrahedron equation. By evaluating their n-fold composition with special boundary vectors we generate series of solutions to the Yang-Baxter equation. Finding their origin in conventional quantum group theory is a clue to the link between two and three dimensional integrable systems. We identify them with the quantum R matrices associated with the q-oscillator representations of $U_q(A^{(2)}_{2n})$, $U_q(C^{(1)}_n)$ and $U_q(D^{(2)}_{n+1})$.

Constructing the Quantum Hall System on the Grassmannians $\mathrm{Gr}_2(\mathbb{C}^N)$

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Joint work with: F.Balli, A.Behtash and G.Unal

In this talk, we will give the formulation of Quantum Hall Effects (QHEs) on the complex Grassmann manifolds $\mathbf{Gr}_2(\mathbb{C}^N)$. We will set up the Landau problem in $\mathbf{Gr}_2(\mathbb{C}^N)$, solve it using group theoretical techniques and provide the energy spectrum and the eigenstates in terms of the $SU(N)$ Wigner $\mathcal{D}$-functions for charged particles on $\mathbf{Gr}_2(\mathbb{C}^N)$ under the influence of abelian and non-abelian background magnetic monopoles or a combination of these thereof. For the simplest case of $\mathbf{Gr}_2(\mathbb{C}^4)$ we will provide explicit constructions of the single and many-particle wavefunctions by introducing the Plücker coordinates and show by calculating the two-point correlation function that the lowest Landau level (LLL) at filling factor $\nu = 1$ forms an incompressible fluid. Relation to matrix models and string physics will also be briefly mentioned. Finally, we will heuristically identify a relation between the $U(1)$ Hall effect on $\mathbf{Gr}_2(\mathbb{C}^4)$ and the Hall effect on the odd sphere S^5, which is yet to be investigated in detail, by appealing to the already known analogous relations between the Hall effects on $\mathbb{C}P^3$ and $\mathbb{C}P^7$ and those on the spheres S^4 and S^8, respectively.

The talk is based on the recent article arXiv:1403.3823 [hep-th] by F.Balli, A.Behtash, S.Kurkcuoglu and G.Unal, "Quantum Hall Effect on the Grassmannians $\mathbf{Gr}_2(\mathbb{C}^N)$," and accepted for publication in **Phys.Rev.D** as a regular article.

Polynomial Symmetries of Classical and Quantum Spherical Lissajous Systems

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Joint work with: J.A. Calzada, J. Negro

A unifying algebraic approach based on factorization method is introduced for finding symmetries of a class of two-dimensional quantum and classical superintegrable systems on the sphere. We will refer to these systems as “spherical lissajous systems”. First, we have found what we call a “fundamental set of symmetries” and then we have obtained the polynomial symmetries. In particular, we have also shown how the factorization properties allow us to solve completely the quantum and classical evolution problems corresponding to mentioned systems.

α-Molecules: Wavelets, Shearlets, and beyond

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Joint work with: P. Grohs (ETH Zurich), S. Keiper (TU Berlin), and M. Schäfer (TU Berlin)

The area of applied harmonic analysis provides a variety of multiscale systems such as wavelets, curvelets, shearlets, or ridgelets, many of those arising from unitary representations of locally compact groups. A distinct property of each of those systems is the fact that it sparsely approximates a particular class of functions. Sparse approximation properties are key to a variety of methodologies such as also for the novel area of compressed sensing. Some of these systems even share similar approximation properties such as curvelets and shearlets which both optimally sparsely approximate functions governed by curvilinear features, a fact that is usually proven on a case-by-case basis for each different construction.

In this talk we will introduce the novel concept of α-molecules which allows for a unified framework encompassing most multiscale systems from the area of applied harmonic analysis with the parameter α serving as a measure for the degree of anisotropy. The main result essentially states that the cross-Gramian of two systems with the same degree of anisotropy exhibits a strong off-diagonal decay. One main consequence we will discuss is that all such systems then share similar approximation properties, and desirable approximation properties of one can be deduced for virtually any other system with the same degree of anisotropy.

Phase amplitude conformal symmetry in Fourier transforms

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Under the Fourier transform $\mathcal{F} : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ of a function $\varphi : \mathbb{R} \to \mathbb{C}$, it is found that for an even or odd function $\varphi(x)$, the condition $|\varphi| = |\mathcal{F}\varphi|$ leads to $\arg \varphi = \pm \arg(\mathcal{F}\varphi)$ up to an additive constant. The converse holds in an analogous way. The condition $|\varphi| = |\mathcal{F}\varphi|$ is required in dealing with, for example, the minimum uncertainty relation between position and momentum. It should be remarked that without the evenness or oddness of φ, the above statement does not hold in general. However, it is conjectured that any $\varphi \in \Psi$ has no zero on the real axis, where $\Psi = \{\varphi : \mathbb{R} \to \mathbb{C} | \arg \varphi \neq \pm \arg(\mathcal{F}\varphi) + \text{const.}, |\varphi| = |\mathcal{F}\varphi|\}$.

N-extended Galilei superconformal algebras and their representations.

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Joint work with: N. Aizawa and F. Toppan

We present a general way of constructing the representations for N-extended (1-dimensional) supersymmetry, extended superconformal algebras and l- Galilei N-extended superalgebras.

As an example we consider the inequivalent $N = 2$ supersymmetrizations of the l-conformal Galilei algebra in d spatial dimensions, which are constructed from the chiral (2,2) and the real (1,2,1) basic supermultiplets of the $N = 2$ supersymmetry. For non-negative integer and half-integer l both superalgebras admit a consistent truncation with a (different) finite number of generators. The real $N = 2$ case coincides with the superalgebra introduced by Masterov, while the chiral $N = 2$ case is a new super algebra.

We present D-module representations of both superalgebras. Then we investigate the new superalgebra derived from the chiral supermultiplet. It is shown that it admits two types of central extensions, one is found for any d and half-integer l and the other only for $d = 2$ and integer l. For each central extension the centrally extended l-superconformal Galilei algebra is realized in terms of its super-Heisenberg subalgebra generators.

Measurement uncertainty relations

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Joint work with: Paul Busch, Reinhard Werner

Measurement uncertainty relations are quantitative bounds on the errors in an approximate joint measurement of two observables. They can be seen as a generalization of the error/disturbance tradeoff first discussed heuristically by Heisenberg. Here we prove such relations for the case of two canonically conjugate observables like position and momentum, and establish a close connection with the more familiar preparation uncertainty relations constraining the sharpness of the distributions of the two observables in the same state.

Emergent symmetries in atomic nuclei from first principles

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Joint work with: A. C. Dreyfuss, R. Baker, J. P. Draayer, T. Dytrych

I will discuss an innovative symmetry-guided approach and its applications to light and intermediate-mass nuclei. This approach, with Sp(3,R) the underpinning group, is based on our recent remarkable finding, namely, we have identified the symplectic Sp(3,R) as an approximate symmetry for low-energy nuclear dynamics. I will present the results of two complementary studies, one that utilizes realistic nucleon-nucleon interactions and unveils symmetries inherent to nuclear dynamics from first principles (or *ab initio*), and another study, which selects important components of the nuclear interaction to explain the primary physics responsible for emergent phenomena, such as enhanced collectivity and alpha clusters. In particular, within this symmetry-guided framework, *ab initio* applications of the theory to light nuclei reveal the emergence of a simple orderly pattern from first principles [1,2]. This provides a strategy for determining the nature of bound states of nuclei in terms of a relatively small fraction of the complete shell-model space, which, in turn, can be used to explore ultra-large model spaces for a description of alpha-cluster and highly deformed structures together with associated rotations. We find that by using only a fraction of the model space extended far beyond current no-core shell-model limits and a long-range interaction that respects the symmetries in play, the outcome reproduces characteristic features of the low-lying 0^+ states in ^{12}C (including the elusive Hoyle state of importance to astrophysics) and agrees with *ab initio* results in smaller spaces [3]. For these states, we offer a novel perspective emerging out of no-core shell-model considerations, including a discussion of associated nuclear deformation, matter radii, and density distribution. The framework we find is also extensible beyond ^{12}C, namely, to the low-lying 0^+ states of ^{8}Be as well as the ground-state rotational band of Ne, Mg, and Si isotopes.

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The Fischer decomposition of polynomials on superspace

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Recently the Fischer decomposition of polynomials on superspace $\mathbb{R}^{m|2n}$ (that is, polynomials in m commuting and $2n$ anti-commuting variables) has been obtained unless the superdimension $M = m - 2n$ is even and non-positive. In this case, it turns out that the Fischer decomposition is an irreducible decomposition under the natural action of Lie superalgebra $osp(m|2n)$. See works of F. Sommen, H. De Bie, D. Eelbode, K. Coulembier and others. In this talk, we describe the Fischer decomposition in the remaining case (that is, when the superdimension is even and non-positive). In particular, we show that, under the action of $osp(m|2n)$, the Fischer decomposition is not, in general, a decomposition into irreducible but indecomposable pieces. This is a joint work with D. Smid and V. Soucek.

Bifractional coherent states

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Joint work with: S. Agyo, A. Vourdas

Bifractional displacement operators, are introduced by performing two fractional Fourier trans- forms on displacement operators. Acting with them on the vacuum we get various classes of coherent states, which we call bifractional coherent states. The uncertainties and the statistical properties of the bifractional coherent states are studied. Bifractional Wigner functions, which in special cases reduce to the Wigner and Weyl functions, are also discussed.

Partial and quasi dynamical symmetries in quantum many-body systems

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The concept of dynamical symmetry (DS) is now widely recognized to be of central importance in our understanding of complex systems. It had major impact on developments in diverse areas of many-body physics, including, hadrons, nuclei, atoms, molecules. Its basic paradigm is to write the Hamiltonian in terms of Casimir operators of a chain of nested algebras. Its hallmarks are the solvability of the complete spectrum, and the existence of exact quantum numbers for all eigenstates. The merits of a DS are self-evident. However, in most applications to realistic systems, the predictions of an exact DS are rarely fulfilled and one is compelled to break it. More often one finds that, in a given system, the assumed symmetry is not obeyed uniformly, *i.e.*, is fulfilled by only some states but not by others. In describing a transition between different structural phases, the relevant Hamiltonian, in general, involves competing interactions with incompatible symmetries. The need to address such situations has led to the introduction of partial dynamical symmetry (PDS) [1] and quasi dynamical symmetry (QDS) [2]. These intermediate-symmetry notions and their implications for dynamical systems, are the subject matter of the present contribution.

The essential idea in a PDS is to relax the stringent conditions of *complete* solvability and exact symmetry, so that they apply to only *part* of the eigenstates and/or of the quantum numbers. The PDS picks particular terms which preserves the DS for a segment of the spectrum and breaks it in the remaining states. In a QDS, particular states continue to exhibit selected characteristic properties of the closet DS, in the face of strong symmetry-breaking interactions. This "apparent" symmetry is due to a coherent mixing of representations in selected states, imprinting an adiabatic motion. Both PDS and QDS are applicable to any many-body system (bosonic and fermionic) system endowed with an algebraic structure.

In the present contribution, we survey the various types of PDS, discuss algorithms for construction of Hamiltonians with this property, including higher-order terms [3,4], and establish a link between PDS and QDS [5]. We present empirical examples and demonstrate the relevance of these notions to nuclear spectroscopy, to quantum phase transitions and to mixed systems with regularity and chaos. The analysis serves to highlight the potential role of PDS and QDS towards understanding the emergent "simplicity out of complexity" exhibited by complex many-body systems.

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Rotations of the most asymmetric molecules via 4-step and 1-step ladder operators

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Kramers and Ittmann pioneered the study of the quantum rotations of asymmetric molecules using Lamé harmonic polynomials [1], and Patera and Winternitz identified the corresponding representations of the rotational group [2]. The exact solutions formulated in [1] could not be implemented numerically for the higher-excitation states, with the consequent development of the perturbation-theory approach for the analysis of the experimental rotational spectra of asymmetric molecules, initiated by Kronig and Rabi [3]. More recently, Piña used the separation of the complete Hamiltonian as the sum of the Hamiltonian for a spherical top and the Hamiltonian of the asymmetry distribution in any molecule [4], obtaining analytical results for some of their properties. In a complementary investigation, Valdés and Piña focused on the most asymmetric molecules characterizing and illustrating their energy levels and eigenfunctions, as well as identifying 4-step ladder operators connecting the states with angular momenta ℓ and $\ell \pm 4$, vanishing asymmetry distribution energy, and negative $y \to -y$ parity [5]. Our own work has been motivated by the need to evaluate accurate and reliable eigenenergies and eigenfunctions of any excitation for molecules of any asymmetry [6]. We have also identified three sets of one-step ladder operators connecting: 1) states with a common value of ℓ , the same symmetry, and neighboring numbers of nodes in the respective elliptical coordinates $n_1 \to n_1 \pm 1$, $n_2 \mp 1$; 2) states with the same value of ℓ, the same kind, and different species; and 3) states with neighboring values of $\ell \to \ell \pm 1$, and different kinds and species, for molecules with any asymmetry [7]. In this contribution we illustrate how the combination of the 4-step operators from [5] and the 1-step operators of [7]-2) are able to connect all the states of the most asymmetric molecules.

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Feynman Diagrams, Representations of U(2,2) and Quaternionic Analysis

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Feynman diagrams are a pictorial way of describing integrals predicting possible outcomes of interactions of subatomic particles in the context of quantum field physics. Some of these diagrams describe integrals that are divergent in mathematical sense, but physicists have different renormalization procedures that "cancel out the infinities" coming from different parts of the diagrams. From a mathematical standpoint, it is highly desirable to have an intrinsic mathematical interpretation of Feynman diagrams.

In this talk I will describe the representation-theoretical meaning of an infinite family of Feynman diagrams. This is done in the context of representations of a Lie group U(2,2) and quaternionic analysis.

No prior knowledge of physics, Feynman diagrams or quaternionic analysis is assumed from the audience.

Nonintersecting paths on the circle and the tacnode process

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Joint work with: Dong Wang

The *tacnode process* is a multi-layer stochastic process which appears as the scaling limit of a model in which two groups of nonintersecting paths just meet at a single point. It is a determinantal process, meaning that the location of particles at different times is described by the determinant of a matrix defined by an *extended kernel*. We consider the tacnode process as a scaling limit of an ensemble of nonintersecting Brownian motions on the unit circle, conditioned to start at a common point at time $t = 0$, and to end at the same common point at the terminal time $t = T$. The correlation kernel for the nonintersecting paths is formulated as a double contour integral of the discrete Gaussian orthogonal polynomials. We will describe the asymptotic evaluation of this kernel proper scaling limit, yielding a formula for the tacnode kernel which involves a particular solution to a 2×2 Lax system for the Painlevé II equation.

Molecular fraction calculation for an atomic-molecular Bose-Einstein condensate model

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Joint work with: Ian Marquette

An integrable model describing the interconversion between Bose-Einstein condensates of atomic and molecular degrees of freedom will be introduced. Through a Bethe ansatz approach, a formula to quantify the average molecular fraction will be derived. Its role in characterising a quantum phase transition of the system will be discussed.

Maximal quantum mechanical symmetry: Projective representations of the inhomogeneous symplectic group

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Physical measurable transition probabilities in quantum mechanics are given by the square of the modulus of the states. A symmetry in quantum mechanics that leaves invariant these physical transition probabilities is described by the projective representation of a Lie group. The Heisenberg commutation relations are the Hermitian representation of the algebra of the Weyl-Heisenberg group corresponding to its unitary representations. The Heisenberg commutation relations must also be valid in any physical state related by the representation of a symmetry group. Otherwise the quantum commutation relations would be valid for some physical states in the Hilbert space and not for others, which would be physically unacceptable. We determine the symmetry group and its projective representations that preserves the Heisenberg commutation relations. We show that these are given by the projective representations of the inhomogeneous symplectic group and that these representations are equivalent to the unitary representations of its central extension. The central extension of the inhomogeneous symplectic group is the cover of the symplectic group semidirect the Weyl-Heisenberg group. These unitary representation are computed explicitly using the Mackey representations for semidirect product groups giving an elegant result. On the other hand, the symmetry of special relativity is the inhomogeneous Lorentz group. Its projective representations, that are the framework of special relativistic quantum mechanics, was originally studied in the seminal work of Wigner. These projective representations of the inhomogeneous Lorentz group make no reference to the Weyl-Heisenberg group that is fundamental to the Heisenberg commutation relations. We discuss the physical consequence of requiring requiring the symmetries of both relativity and Heisenberg commutation relations. J. Math. Phys. 55 022105 (2014) or arXiv:1207/6787

On the boundaries of quantum integrability for the spin-$1/2$ Richardson–Gaudin system

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Joint work with: Phillip Isaac, Jon Links

We investigate the quasi-classical limit of Sklyanin's boundary quantum inverse scattering method, which leads us to Richardson-Gaudin type models. They constitute an important class of quantum integrable models related to the BCS theory of superconductivity. Remarkable properties of the Bethe Ansatz equations, conserved operators and their eigenvalues are unveiled in this approach.

New families of superintegrable systems from k-step rational extensions, polynomial algebras and degeneracies

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Joint work with: Christiane Quesne , Universite Libre de Bruxelles, Belgium

I will discuss recent results on new families of 2D Superintegrable systems possessing higher order integrals of motion and involving k-step extension of harmonic and radial oscillator. Their wavefunctions are related with exceptional orthogonal polynomials. I will discuss how from the polynomial algebras, we can obtain the finite dimensional unitary representations via the Daskaloyannis approach and also using a more direct approach. From these results an algebraic derivation of the whole energy spectrum and the total number of degeneracies can be done. These systems also exhibit more complicated patterns for the degeneracies than usual 2D isotropic harmonic oscillator.

Talk based on :

I.Marquette and Christiane Quesne, Combined state-adding and state-deleting approaches to type III multi-step rationally-extended potentials: applications to ladder operators and superintegrability, arXiv:1402.6380

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Connections and Euler-Lagrange equations on Lie algebroids

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Lie algebroides provide a general and appropriate framework for studying Lagrangian and Hamiltonian mechanics for standard systems, for systems with symmetry and other more general systems. The Euler-Lagrange equations for a Lagrangian system on a Lie algebroid will be expressed in terms of a linear connection. The particular case of Poincaré equations when the Lie algebroid is just a Lie algebra will be studied and interpreted as the equation for parallel transport of the momentum.

On symmetries and recursion operators of the Darboux–Egoroff system

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Joint work with: Sergei Igonin

The Darboux–Egoroff system

$$
\begin{aligned}
h_{ij,j} &= -h_{ij,i} - \sum_s h_{is}h_{sj}, \qquad i < j, \\
h_{ij,k} &= h_{ik}h_{kj}, \qquad i, j, k \text{ pairwise different,}
\end{aligned}
$$

in unknowns $h_{ij} = h_{ji}$, $h_{ii} = 0$, $i = 1, \dots, n$, with any number $n > 2$ of independent variables, plays an essential role in the description of flat diagonal metrics of Egoroff type and Frobenius manifolds. Applying a general procedure to the known zero curvature representation, we compute the recursion operator and its inverse. The recursion operator acts as follows: If H_{ij} are components of a symmetry, then so are

$$
\begin{aligned}
\mathfrak{R}(H)_{ij} = H_{ij,ii} + h_{ij,i}P_i - \Big(h_{ij,i} + \sum_s h_{is}h_{sj}\Big)P_j& \\
+ \sum_s \Big(h_{is}H_{sj,s} + h_{is}^2 H_{ij} + 3h_{ij}h_{is}H_{is}& \\
+ \Big(h_{is,i} - h_{ij}h_{sj} + \sum_r h_{ir}h_{sr}\Big)H_{sj}\Big)& \\
+ \sum_s h_{is}h_{sj}P_s, \quad i \neq j,&
\end{aligned}
$$

where P_i are determined by the system

$$
P_{i,k} = \begin{cases} -2\sum_s h_{is}H_{is}, & i = k, \\ 2h_{ik}H_{ik}, & i \neq k. \end{cases}
$$

The third- and fifth-order symmetries generated from the zero seed contain the third- and fifth-order flows

$$
\begin{aligned}
u_{t_3} &= u_{xxx} + 3(u,u)u_x + 3(u,u_x)u, \\
u_{t_5} &= u_{xxxxx} + 5(u,u)u_{xxx} + 15(u,u_x)u_{xx} + 15(u,u_{xx})u_x + 10(u_x,u_x)u_x \\
&\quad + 10(u,u)^2 u_x + 5(u,u_{xxx})u + 10(u_x,u_{xx})u + 20(u,u)(u,u_x)u
\end{aligned}
$$

of the $(n-1)$-component vector mKdV hierarchy. The inverse recursion operator provides a way to generate infinitely many nonlocal symmetries.

For details, see `arXiv:1403.6109 [nlin.SI]`.

Homogeneity and Lagrangian systems

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In this talk we show that the unparametrized trajectories of a Lagrangian system, after a suitable choice of parameter, can be derived from a variational principle involving a 1-homogeneous function on the tangent bundle. Our approach is based on Routh's reduction procedure for symmetry Lie groups. We will also discuss the so-called Mane critical value in this context. This value of the total energy provides a lower bound for the 1-homogeneous function to be in fact a Finsler function.

Projective families of Dirac operators on a Banach Lie groupoid

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Joint work with: Pedram Hekmati

We introduce a Banach Lie group G of unitary operators subject to a natural trace condition. We compute the homotopy groups of G, describe its cohomology and construct an S^1-central extension. We show that the central extension determines a non-trivial gerbe on the action Lie groupoid $G \ltimes \mathfrak{k}$, where $\mathfrak{k}$ denotes the Hilbert space of self-adjoint Hilbert–Schmidt operators. With an eye towards constructing elements in twisted K-theory, we prove the existence of a cubic Dirac operator $\mathbb{D}$ in a suitable completion of the quantum Weil algebra $\mathcal{U}(\mathfrak{g}) \otimes Cl(\mathfrak{k})$, which is subsequently extended to a projective family of self-adjoint operators $\mathbb{D}_A$ on $G \ltimes \mathfrak{k}$. While the kernel of $\mathbb{D}_A$ is infinite-dimensional, we show that there is still a notion of finite reducibility at every point, which suggests a generalized definition of twisted K-theory for action Lie groupoids.

Crossed products for non-commutative Poisson algebras. Applications

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Joint work with: Ana Agore

Poisson algebras are important objects of study for quantum groups. The crossed products of non-commutative Poisson alebras are introduced as the toll for the study of the the global extension problem (GEP) at the lavel of Poisson algebras which consists of the following question: let P be a Poisson algebra, E a vector space and $\pi : E \to P$ an epimorphism of vector spaces with $V = \mathrm{Ker}(\pi)$. The GEP asks for the classification of all Poisson algebra structures that can be defined on E such that $\pi : E \to P$ becomes a morphism of Poisson algebras. From geometrical point of view it means to decompose this groupoid into connected components and to indicate a point in each such component. Several examples are provided in the case of metabelian Poisson algebras or co-flag Poisson algebras over P: the latter being Poisson algebras Q which admit a finite chain of epimorphisms of Poisson algebras $P_n := Q \xrightarrow{\pi_n} P_{n-1} \cdots P_1 \xrightarrow{\pi_1} P_0 := P$ such that $\dim(\mathrm{Ker}(\pi_i)) = 1$, for all $i = 1, \cdots, n$.

The theory of contractions of 2D 2nd order quantum superintegrable systems

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Joint work with: E. Kalnins Waikato), S. Post (Hawaii) , E. Subag (Tel Aviv) and R. Heinonen (Minnesota)

We describe a contraction theory for 2nd order superintegrable systems, showing that all such systems in 2 dimensions are limiting cases of a single system: the generic 3-parameter potential on the 2-sphere. Analogously, all of the quadratic symmetry algebras of these systems can be obtained by a sequence of contractions starting from the generic system. By contracting function space realizations of irreducible representations of the generic symmetry algebra (which give the structure equations for Racah/Wilson polynomials) to the other superintegrable systems one obtains the full Askey scheme of orthogonal hypergeometric polynomials. This relates the scheme directly to explicitly solvable quantum mechanical systems. Amazingly, all of these contractions of superintegrable systems with potential are uniquely induced by Wigner Lie algebra contractions.

Hecke Graphs, Ramanujan Graphs and Generalized Duality transformations for Lattice Spin Systems

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I discuss two related subjects: 1) Hecke surfaces and K regular graphs, 2) duality transformations for general Potts models. Each of them is related to deep mathematical and physical theories, and at a first glance, they have nothing in common. However, it became evident in recent years that there exist deep internal relations between these two problems. Especially interesting (and mysterious) is the role of Hecke groups in this context.

I consider a few examples of relevant topics:

A. Hecke surfaces with large cusps.

B. Hecke graphs and Ramanujan graphs.

C. Potts models and the cluster behavior of the zeros of the chromatic polynomials on lattices.

Supersymmetric partners of the truncated harmonic oscillator

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Joint work with: David J. Fernández C.

Supersymmetry transformations of first and second order are used to generate Hamiltonians with known spectra departing from the harmonic oscillator with an infinite potential barrier. Certain systems obtained in a straightforward way through said procedure possess differential ladder operators of both types, third and fourth order. Since systems with this kind of operators are linked with the Painlevé IV and Painlevé V equations respectively, several solutions of these non-linear second-order differential equations will be simply found, along with a chain of Bäcklund transformations connecting such solutions.

Integrability in the dimer model

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Joint work with: Jorgen Rasmussen, Philippe Ruelle

The dimer model on the square lattice is a statistical model that can be investigated using a transfer matrix $T(\alpha)$, where α is the weight assigned to horizontal dimers. It is exactly solvable in the sense that the transfer matrix can be rewritten in terms of non interacting fermions, which allows for an exact computation of the eigenvalues of $T(\alpha)$ and of the partition functions for all lattice sizes. However, because $[T(\alpha), T(\beta)] \neq 0$ in general, the dimer model does not satisfy the usual criteria for Yang-Baxter integrability. In this talk, we discuss an intrinsic relation between the dimer model and critical dense polymers, a Temperley-Lieb loop model with loop fugacity $\beta = 0$, and use this to reconciliate free-fermion and Yang-Baxter integrability for the dimer model.

Dynamical Formulation of Scattering Theory and Unidirectional Invisibility

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For a given real or complex scattering potential v(x), we construct a non-stationary and non-Hermitian two-level Hamiltonian H(t) whose S-matrix coincides with the transfer matrix of v(x). We use this approach to develop a dynamical theory of scattering where the reflection and transmission amplitudes are given as solutions of a set of dynamical equations, offer an inverse scattering scheme for constructing scattering potentials with desirable properties at a prescribed wavelength, and give a perturbative analysis of the phenomenon of unidirectional invisibility. In particular, we construct multimode unidirectionally invisible potentials with wavelength-dependent direction of invisibility. If time allows, we also report our results on the equivalence of the adiabatic approximation for H(t) and the semiclassical (WKB) approximation for v(x), comment on the interesting role of geometric phases in this context, and show how one can use the adiabatic series expansion for the evolution operator of H(t) to outline higher order WKB approximations for v(x).

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Coherent states quantization and formulae for the Berezin transform.

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Abstract: We are concerned with bound states spaces of the Schrodinger operator with magnetic field in Cn and in the complex unit ball Bn (eigenspaces corresponding to Euclidean Landau levels and hyperbolic Landau levels respectively). We construct for each of these spaces a set of coherent states to apply a coherent states quantization method. This enables us to recover the Berezin transforms attached to these spaces. In each case, we give formulae representing these transforms as functions of the Laplace-Beltrami operator.

Localization and q-deformations of Lie algebras and superalgebras

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In a recent work (P. Moylan, J. Phys. Conf. Ser. 512 012026 (2014)) we described an extension of $U_q(\mathfrak{osp}(1|2))$ by means of an algebraic extension of a localization of the algebra $U(H)$ where H is the basic Cartan generator for $U_q(\mathfrak{osp}(1|2))$. There we also considered a similar extension for $U_q(\mathfrak{sl}(2))$, and we constructed explicit homomorphisms of $\mathfrak{iso}(2)$ into these extensions and also described related homomorphism of $U_q(\mathfrak{osp}(1|2))$ and $U_q(\mathfrak{sl}(2))$ into extensions of $U(\mathfrak{iso}(2))$. Such homomorphisms enable us to obtain new representations of $U_q(\mathfrak{osp}(1|2))$ and $U_q(\mathfrak{sl}(2))$ out of representations of $\mathfrak{iso}(2)$. Here we further these results and consider generalizations to q deformations of higher dimensional Lie algebras and superalgebras.

Logarithmic knot invariants coming from the restricted quantum groups

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By using the centers in the Jacobson radical of the restricted quantum group $\overline{\mathcal{U}}_q(sl_2)$ at root of unity, the logarithmic invariants of knots are introduced by Nagatomo and me. Here I give a formula for the logarithmic invariant in terms of the colored Jones invariant, and investigate its relation to the hyperbolic volume. Let $\xi = \exp(\pi\sqrt{-1}/N)$, then the center of $\overline{\mathcal{U}}_\xi(sl_2)$ is $3N-1$ dimensional, and its good basis

$$\{\hat{\boldsymbol{\rho}}_1, \cdots, \hat{\boldsymbol{\rho}}_{N-1}, \hat{\boldsymbol{\varphi}}_1, \cdots, \hat{\boldsymbol{\varphi}}_{N-1}, \hat{\boldsymbol{\kappa}}_0, \cdots, \hat{\boldsymbol{\kappa}}_N\}$$

is given by Feigin-Gainutdinov-Semikhatov-Tipunin which behaves well under certain action of $SL(2,\mathbb{Z})$. For a knot L, let $\gamma_s(L)$ be the logarithmic invariant corresponding to $\hat{\boldsymbol{\kappa}}_s$, and let $V_\lambda(L)$ be the colored Jones invariant for the $\lambda+1$ dimensional representation of $\mathcal{U}_q(sl_2)$ at generic q. Then the following formula holds.

Theorem *The invariant $\gamma_s(L)$ is given by*

$$\gamma_s(L) = \frac{\xi}{2N}\,\frac{d}{dq}(q-q^{-1})\big(V_{s-1}(L) + V_{2N-s-1}(L)\big)\Big|_{q=\xi}.$$

On the other hands, we have the following conjecture, which is first noticed by Kashaev.

Conjecture (Kashaev's conjecture). Let L be a hyperbolic knot in S^3. Then

$$\lim_{N\to\infty} \frac{2\,\pi\,\log|J_N(L)|}{N} = \mathrm{Vol}\left(\mathrm{S}^3 \setminus \mathrm{L}\right),$$

where $\mathrm{Vol}\left(\mathrm{S}^3 \setminus \mathrm{L}\right)$ is the hyperbolic volume of $S^3 \setminus L$.

Our invariant $\gamma_s(L)$ can be considered as a deformation of $J_N(L)$ since $J_N(L)$ is equal to $\gamma_N(L)$, and I propose the following conjecture.

Conjecture (Volume conjecture for the logarithmic invariant). Let L be a hyperbolic knot and let M_α be the cone manifold along the singularity set L with the cone angle α. Let s_N be a sequence of integers such that $\lim_{N\to\infty}\frac{s_N}{N} = \frac{\alpha}{2\,\pi}$. If M_α is a hyperbolic manifold, then

$$\lim_{N\to\infty} \frac{2\,\pi\log\left|\gamma_{s_N}(L)\right|}{N} = \mathrm{Vol}(\mathrm{M}_\alpha).$$

For the figure-eight knot, this conjecture is proved for α satisfying $0 \le \alpha < \frac{\pi}{3}$ and $\frac{5\pi}{3} < \alpha \le 2\,\pi$, and is checked numerically for entire α.

At the end of this talk, I would like to introduce generalized logarithmic knot invariant corresponding to a restricted quantum group of higher rank.

Shapovalov elements for basic simple Lie superalgebras

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We provide upper bounds on the degrees of the coefficients of Šapovalov elements for a simple Lie algebra. If $\mathfrak{g}$ is a contragredient Lie superalgebra and γ is a positive isotropic root of $\mathfrak{g}$, we prove the existence and uniqueness of the Šapovalov element for γ and we obtain upper bounds on the degrees of their coefficients. For type A Lie superalgebras we give a closed formula for Šapovalov elements. We also explore the behavior of Šapovalov elements coming when the Borel subalgebra is changed, and the survival of Šapovalov elements in factor modules of Verma modules.

Reflection positivity and unitary representations of Lie groups

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Joint work with: Gestur Olafsson, Palle Jorgensen

Reflection positivity (sometimes called Osterwalder-Schrader positivity) was introduced by Osterwalder and Schrader in the context of axiomatic euclidean field theories. On the level of unitary representations, it provides a passage from representations of the euclidean isometry group to representations of the Poincaré group. In our talk we shall explain how these ideas can be used to obtain a natural context for the passage from representations of Lie groups with an involutive automorphism (symmetric Lie groups) to representations of their dual Lie group. Already the case of one-parameter groups is of considerable analytic interest. For more general Lie groups this passage requires new results on integrability of representations of Lie algebras. Another challenging aspect is the relation to stochastic processes indexed by Lie groups.

Superintegrable anisotropic oscillators on two-dimensional spheres and hyperboloids

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It is proposed a class of superintegrable anisotropic oscillators systems defined on the two-dimensional sphere and hyperboloid. They are described by parallel geodesic coordinates that in the limit of zero curvature turn into cartesian coordinates. The symmetries and constants of motion are computed for the quantum and classical versions.

A Group Theoretical approach to Quantum Theory of Interaction

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The basic problems in establishing the Quantum Theory of a specific physical system are

(1) to single out, *modulo* unitary isomorphism, the specific Hilbert space $\mathcal{H}$ of the specific theory;

(2) to concretely identify the specific operator of $\mathcal{H}$ representing each specific relevant observable;

(3) to determine the dynamical law, e.g. equation of motion, of the specific theory.

Group theoretical methods developed by outstanding researchers, such as E. Wigner, G. Mackey, allowed to give these problems very satisfactory answers.

E. Wigner proved that the symmetry group G of the system must have a projective representation $G \ni g \to U_g$ in the Hilbert space $\mathcal{H}$ of its Quantum Theory.

Now, if the physical system is a free localizable particle, then Galilei's group $\mathcal{G}$ is a symmetry group. On the other hand, the natural covariance properties imply that the position operators $\mathbf{Q} = (Q_x, Q_y, Q_z)$ form an Imprimitivity System. Therefore Mackey's theorem applies, and in so doing problems (1), (2) and (3) are solved in full logical, mathematical and conceptual rigors for the Quantum Theory of a free galileian particle.

Unfortunately, if the system is not isolated, generally Galilei's transformations $g \in \mathcal{G}$ are *not* symmetries, so that Wigner's theorem does not apply. This is the first obstacle in trying to establish the Quantum theory of an interacting system following a Group Theoretical approach.

The formulations of the currently practised interaction theories developed by making use of Gauge Principle introduced by Yang and Mills, despite of their success, suffer epistemological defects. For instance, it is not explained the origin of the prescriptions for obtaining the dynamical law of the non-interacting system.

In this work we address the problem of establishing the general Quantum Theory of an interacting particle following the Group Theoretical approach. In so doing we encounter the problem that the transformations group $\mathcal{G}$ is not a symmetry group and therefore $\mathcal{G}$ does not immediately have a projective representation in $\mathcal{H}$; moreover, the position operators do not form an Imprimitivity System. We overcome these obstacles by some theoretical devices. One of them consists in adopting the *passive* interpretation of the transformations of $\mathcal{G}$, rather than the active interpretation adopted in the non-interacting case. Furthermore, we devise an alternative way of making use of Imprimitivity Theorem which applies in our approach.

As a result, we find how the Quantum Theory of an interacting particle can be established with epistemological soundness, and how different possibilities for the form of the interaction can be characterized in Group Theoretical terms.

Meromorphic solutions to the q-Painlevé equations around the origin

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We study special solutions to the q-Painlevé equations with type VI, V, III. We assume that $0 < |q| < 1$. We set $\bar{f} = f(qt)$ for $f = f(t)$.

$$q\text{-}P_{\mathrm{VI}}: \quad \frac{y\bar{y}}{a_3a_4} = \frac{(\bar{z}-b_1t)(\bar{z}-b_2t)}{(\bar{z}-b_3)(\bar{z}-b_4)}, \quad \frac{z\bar{z}}{b_3b_4} = \frac{(y-a_1t)(y-a_2t)}{(y-a_3)(y-a_4)}, \quad \frac{b_1b_2}{b_3b_4} = q\frac{a_1a_2}{a_3a_4}.$$

$$q\text{-}P_{\mathrm{V}}: \quad \frac{y\bar{y}}{a_3a_4} = -\frac{(\bar{z}-b_1t)(\bar{z}-b_2t)}{\bar{z}-b_3}, \quad \frac{z\bar{z}}{b_3} = -\frac{(y-a_1t)(y-a_2t)}{a_4(y-a_3)}, \quad \frac{b_1b_2}{b_3} = q\frac{a_1a_2}{a_3a_4}.$$

$$q\text{-}P_{\mathrm{III}}: \quad \frac{y\bar{y}}{a_3a_4} = -\frac{\bar{z}(\bar{z}-b_2t)}{\bar{z}-b_3}, \quad \frac{z\bar{z}}{b_3} = -\frac{y(y-a_1t)}{a_4(y-a_3)}.$$

Here $y = y(t)$ and $z = z(t)$ are unknown functions and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are complex parameters. It is known that the q-Painlevé equations are obtained by connection preserving deformations.

Theorem *For generic parameters, the q-P_{VI}, q-P_{V} and q-P_{III} have four, three and two meromorphic solutions around the origin. We can solve linear connection problems for holomorphic solutions.*

For example, the q-P_{V} has a holomorphic solution around the origin:

$$y(t) = (a_3 - a_4b_3) + O(t), \qquad z(t) = \left(b_3 - \frac{a_3}{a_4}\right) + O(t).$$

The linear q-difference equation reduces to the basic hypergeometric equation $Y(xq) = A(x)Y(x)$, where

$$A(x) = \begin{pmatrix} \frac{a_1a_2(b_1+b_2)}{b_1b_2} & a_3\left(\frac{{a_4}^2b_1b_2}{a_1a_2q} - 1\right) \\ \frac{{a_1}^3{a_2}^3q}{a_3b_1b_2(a_1a_2q-a_4^2b_1b_2)} & 0 \end{pmatrix} + \xi \begin{pmatrix} -\frac{a_1a_2}{a_4b_1b_2} & 0 \\ \frac{a_1^2a_2^2(a_1+a_2-a_4b_1-a_4b_2)q}{a_3b_1b_2(a_1a_2q-a_4^2b_1b_2)} & -a_4 \end{pmatrix}.$$

The solution of the reduced linear equation is given by ${}_2\varphi_1\left(\frac{a_4b_2}{a_2}, \frac{a_1q}{a_4b_1}; \frac{b_2q}{b_1}; \frac{\xi}{a_1}\right)$.

Unique characterization of the Fourier transform in the framework of representation theory

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Joint work with: Hendrik De Bie, Joris Van der Jeugt

The Fourier transform (FT) is of crucial importance in a whole range of areas such as harmonic analysis and signal processing as it has many interesting properties.

A natural question is: Which properties are sufficient to uniquely characterize the FT? Additionally, given a specific set of properties of the FT, one can inquire whether there are any other transforms that satisfy these properties. By definition, transforms devised in this way automatically have some favourable properties, which makes them interesting objects on their own, prone to further study.

These questions lead to the following natural course of actions. First, we start from a set of favourable properties of the FT and determine the class of all transforms for which these properties hold. Next, we investigate how we can reduce this class of solutions by imposing additional properties. Ultimately, in this way we want to arrive at a list of properties, the combination of which is exclusively satisfied by the FT. This gives us a unique characterization of the FT.

In order to do this, we work in the framework of representation theory of the Lie algebra $\mathfrak{sl}_2$, and its refinement the Lie superalgebra $\mathfrak{osp}(1|2)$. The reason for this is that the FT can be expressed as an operator exponential with operators that generate a realization of $\mathfrak{sl}_2$. Moreover, there is also a direct relation between these operators and the properties we want to impose.

The natural generalization to the Lie superalgebra $\mathfrak{osp}(1|2)$ brings us to the setting of Clifford analysis. In doing so, we obtain Clifford algebra-valued transforms that act on functions taking values in a Clifford algebra. Furthermore, the Clifford structure allows us to further refine the imposed properties to more strict properties.

Darboux transformation with Dihedral reduction group

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We present a Darboux transformation with Dihedral reduction group for the 2 - dimensional generalisation of the periodic Volterra lattice. The resulting Bäcklund transformation can be viewed as a nonevolutionary integrable differential difference equation. We also find its generalised symmetry and the Lax representation for this symmetry.

Affine Kac-Moody Symmetric spaces associated with untwisted Kac-Moody algebras

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Symmetric spaces associated with Lie algebras and Lie groups which are Riemannian manifolds have recently got a lot of attention in various branches of Physics for their role in classical/ quantum integrable systems, transport phenomena etc. Their infinite dimensional counterpart have recently been discovered called affine Kac-moody Symmetric spaces which are actually tame Frechet manifold. In this talk we have computed all the affine Kac-Moody Symmetric Spaces associated with untwisted Kac-Moody algebras starting from their Vogan diagrams. These diagrams are nothing but modified Dynkin diagrams. In a given table we have provided all the affine Kac-Moody symmetric spaces(compact/non-compact) associated with these algebras along with their fixed point algebras. To corroborate our technique we have explicitly computed these type of spaces for $\mathrm{A}_1^{(1)}, A_2^{(1)}$.

A quantum field theoretical approach to the P vs. NP problem via the sign of the fermionic Quantum Monte Carlo

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I present here a new method that allows the introduction of a discrete auxiliary symmetry in a theory in such a way that the eigenvalue spectrum of the fermion functional determinant is made up of complex conjugated pairs. The method implies a particular way of introducing and integrating over auxiliary fields related to a set of artificial shift symmetries. Gauge-fixing the artificial continuous shift symmetries in the direct and dual sector leads to the implementation of a Kahler structure over the field space. The discrete symmetry appears to be induced by the Hodge-* operator. The particular extension of the field space presented here makes the operators of the de-Rham cohomology manifest. This method implies the identification of the (anti)-BRST and dual-(anti)-BRST operators with the exterior derivative and its dual in the context of the complex de-Rham cohomology. The novelty of this method relies on the fact that the field structure is doubled two times in order to make use of a supplemental symmetry prescribed by algebraic geometry. This leads to a generalization of Kramers theorem that avoids the Quantum Monte Carlo phase sign problem without any apparent increase in complexity.

Allan I. Solomon: Some highlights of his work in Mathematical Physics

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Allan I. Solomon, an eminent Mathematical Physicist and one of the most active members of Standing Committee assuring the existence and continuity of current series of conferences on Group Theoretical Methods in Physics, has died in 2013. An attempt will be made to highlight some of his contributions to Condensed Matter Theory, Quantum Information Theory and Combinatorial Physics.

Superintegrable Systems on the Three-Dimensional Hyperboloids

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In this note the some known super integrable systems on three-dimensional hyperboloids, namely: H_3^1: $z_0^2 - z_1^2 - z_2^2 - z_3^2 = R^2$, (two-sheeted hyperboloid), H_1^3: $z_0^2 - z_1^2 - z_2^2 - z_3^2 = -R^2$ (one-sheeted hyperboloid) and H_2^2: $z_0^2 + z_1^2 - z_2^2 - z_3^2 = R^2$ (Anti de Sitter Space) are discussed. The work done in collaboration with D.Petrosyan.

On Kostant's theorem for the Lie superalgebra $Q(n)$

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A finite W-algebra is a certain associative algebra attached to a pair $(\mathfrak{g}, e)$, where $\mathfrak{g}$ is a complex semisimple Lie algebra and $e \in \mathfrak{g}$ is a nilpotent element. It is a result of B. Kostant that for a regular nilpotent element e, the finite W-algebra coincides with the center of the universal enveloping algebra $U(\mathfrak{g})$.

In the full generality the finite W-algebras were introduced by A. Premet. His definition makes sense for classical Lie superalgebras. However, Kostant's theorem does not hold in this case. Finite W-algebras for Lie superalgebras have been extensively studied by C. Briot, E. Ragoucy, J. Brundan, J. Brown, S. Goodwin, W. Wang, L. Zhao and other mathematicians and physicists.

We study finite W-algebras for basic classical Lie superalgebras and the queer Lie superalgebra $Q(n)$ associated with the regular even nilpotent coadjoint orbits. We show that this finite W-algebra satisfies the Amitsur-Levitzki identity and therefore all its irreducible representations are finite-dimensional. In the case of $Q(n)$ we give an explicit description of the finite W-algebra in terms of generators and relations and realize it as a quotient of the super-Yangian of $Q(1)$. This is a joint work with V. Serganova.

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The classification of superintegrable systems and contractions of quadratic algebras

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Joint work with: J. Kress and J. Capel

In this talk, I will discuss recent advances in understanding the classification of superintegrable systems and their connection to orthogonal polynomials, as interbasis expansion coefficients as well as representations of the associated symmetry algebras. I will review the classification of 2D systems and the connection to limits within the Askey scheme of orthogonal polynomials. I will then present recent work on the 3D analogs and a possible extension of the Askey scheme to multivariable orthogonal polynomials.

Ladder operators for solvable potentials connected with exceptional orthogonal polynomials

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Joint work with: Ian Marquette

Exceptional orthogonal polynomials constitute the main part of the bound-state wavefunctions of some solvable quantum potentials, which are rational extensions of well-known shape-invariant ones. The former potentials are most easily built from the latter by using higher-order supersymmetric quantum mechanics (SUSYQM) or Darboux method. They may in general belong to three different types (or a mixture of them): types I and II, which are strictly isospectral, and type III, for which k extra bound states are created below the starting potential spectrum. A well-known SUSYQM method enables one to construct ladder operators for the extended potentials by combining the supercharges with the ladder operators of the starting potential. The resulting ladder operators close a polynomial Heisenberg algebra (PHA) with the corresponding Hamiltonian. In the special case of type III extended potentials, for this PHA the k extra bound states form k singlets isolated from the higher excited states. Some alternative constructions of ladder operators will be reviewed. Among them, there is one that combines the state-adding and state-deleting approaches to type III extended potentials (or so-called Darboux-Crum and Krein-Adler transformations) and mixes the k extra bound states with the higher excited states. Ian Marquette will show how this novel approach can be used for building integrals of motion for two-dimensional superintegrable systems constructed from rationally-extended potentials.

Higher symmetries of the conformal Laplacian

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Joint work with: J.-P. Michel and J. Silhan

In this talk, we will study the symmetries and the conformal symmetries of the conformal Laplacian on an arbitrary pseudo-Riemannian manifold (M, g).

On an m-dimensional pseudo-Riemannian manifold (M, g), this operator, which we denote here by Δ_Y, is given by

$$\mathrm{g}^{ij}\nabla_i\nabla_j - \frac{m-2}{4(m-1)}R,$$

where ∇ denotes the Levi-Civita connection of g and R its scalar curvature.

A symmetry of Δ_Y is a differential operator which commutes with Δ_Y. A conformal symmetry of Δ_Y is a differential operator D_1 such that there exists a differential operator D_2 giving rise to the relation $\Delta_Y D_1 = D_2\Delta_Y$.

These (conformal) symmetries were completely described on a conformally flat manifold thanks to the works of M. G. Eastwood and J.-P. Michel. In the curved setting, a second-order symmetry of the conformal Laplacian on an Einstein manifold was known thanks to a work of B. Carter.

We will describe in this talk all the second-order (conformal) symmetries of Δ_Y on an arbitrary pseudo-Riemannian manifold (M, g). The principal symbol of such a (conformal) symmetry has to be a symmetric (conformal) Killing 2-tensor that satisfies some additional condition.

We will determine whether this condition is verified on some pseudo-Riemannian manifolds endowed with some (conformal) Killing tensors, determining in this way whether there exists an obstruction to the existence of (conformal) symmetries in these particular situations.

At the end of the talk, we will show how the study of the conformal symmetries (resp. symmetries) of the conformal Laplacian is related to the study of the R-separation of variables in the Laplace (resp. Helmholtz) equation.

Higher spin operators as generators of transvector algebras

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Joint work with: David Eelbode, University of Antwerp

In higher spin Clifford analysis, any irreducible representation of the Spin(m)-group with a half-integer highest weight $\left(l_1 + \frac{1}{2}, \ldots, l_k + \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}\right)$, where $l_1, \ldots, l_k$ are natural numbers, can be modelled by the space of simplicial monogenic polynomials in k vector variables $u_1, \ldots, u_k$, homogeneous of degree l_i in u_i for each $i \in \{1, \ldots, k\}$. This space is denoted by $\mathcal{S}_{l_1,\ldots,l_k}$, see [1].
The theory of generalised gradients (e.g. [2, 4]) tells us that the only conformally invariant first order differential operators acting functions with values in $\mathcal{S}_{l_1,\ldots,l_k}$ (which can be identified with the space of polynomials $\mathcal{C}^\infty(\mathbb{R}^m, \mathcal{S}_{l_1,\ldots,l_k})$) are the higher spin Dirac operator $\mathcal{Q}_{l_1,\ldots,l_k}$, at most k twistor operators, and at most $k+1$ dual twistor operators.
In this talk, it will be shown that these differential operators can be seen as generators of a transvector algebra, hereby generalising the fact that the classical Dirac operator and its symbol generate the orthosymplectic Lie algebra $\mathfrak{osp}(1,2)$. To that end, we construct these operators using an extremal projector, an object that is naturally appearing in the theory of transvector algebras (see e.g. [3, 5, 6]).

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Universality and time-scale invariance for the shape of planar Lévy processes

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For a broad class of planar Markov processes, viz. Lévy processes satisfying certain conditions (valid *eg* in the case of Brownian motion and Lévy flights), we establish an exact, universal formula describing the shape of the convex hull of sample paths. We show indeed that the average number of edges joining paths' points separated by a time-lapse $\Delta\tau \in [\Delta\tau_1, \Delta\tau_2]$ is equal to $2\ln(\Delta\tau_2/\Delta\tau_1)$, regardless of the specific distribution of the process's increments and regardless of its total duration T. The formula also exhibits invariance when the time scale is multiplied by any constant.
Apart from its theoretical importance, our result provides new insights regarding the shape of two-dimensional objects (*eg* polymer chains) modelled by the sample paths of stochastic processes generally more complex than Brownian motion. In particular for a total time (or parameter) duration T, the average number of edges on the convex hull ("cut off" to discard edges joining points separated by a time-lapse shorter than some $\Delta\tau < T$) will be given by $2\ln\left(\frac{T}{\Delta\tau}\right)$. Thus it will only grow logarithmically, rather than at some higher pace.

Exactly solvable model of critical dense polymers

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A two-dimensional lattice model of critical dense polymers is presented. It is the first member $\mathcal{LM}(1,2)$ of the Yang-Baxter integrable series of logarithmic minimal models, and is exactly solvable for arbitrary finite sizes on the strip, cylinder and torus. It is a loop model where contractible loops are disallowed, but depending on the geometry, non-contractible or boundary loops may appear. Using the corresponding (extended) Temperley-Lieb algebra, commuting transfer matrices are formed by acting on link states. These transfer matrices satisfy a functional equation in the form of an inversion identity which is key to obtaining the exact solution. In the continuum scaling limit, the model gives rise to a logarithmic conformal field theory with central charge $c = -2$, and the torus partition function yields a modular invariant. The only comparable model solved in such detail is the two-dimensional Ising model.

Second-order symmetry operators and separation of variables for Dirac equation with external fields on two-dimensional spin manifolds

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Joint work with: L. Fatibene, R.G. McLenaghan

The second-order symmetry operators of the Dirac equation with external fields on two-dimensional spin manifolds are computed. It is shown how they are necessary to characterize in full generality the multiplicative separation of variables of the Dirac equation, extending the existing separation of variables theory that involves first-order symmetry operators only. Several examples are given in Liouville and Polar coordinates.

Lagrangian reductions and integrable systems in condensed matter

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We consider a general approach for the process of Lagrangian and Hamiltonian reduction by symmetries in chiral gauge models. This approach is used to show the complete integrability of several one dimensional texture equations arising in liquid Helium phases and neutron stars.

Harmonic functions for unitary representations of three dimensional Lie groups on their parameter space

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Three dimensional Lie groups share some exceptional properties which enable us to obtain all unitary representations of the Lie groups $SL(2,\mathbb{C}), SU(2), SL(2,\mathbb{R})$ and E_2 as harmonic functions on their parameter space (or eventually on a suitable covering space). It is further shown that all these realisations are related by natural operations as real forms or Inönü-Wigner contractions.

Finite groups and nonassociativity in vertex algebras

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Joint work with: T. De Medts, O. Gray, J. I. Hall, S. Shpectorov

We report on a programme to study finite groups and vertex (operator) algebras via Griess-like algebras, following results by Miyamoto, Dong-Li-Mason-Norton, Matsuo and others. A simple axiomatisation of some nonassociative algebras, based on idempotents with fusion rules, gives results along the lines of those seen in vertex algebras. We present a classification of algebras with fusion rules coming from only two highest weights, which leads to two cases: finite 3-transposition groups as Matsuo algebras, and Jordan algebras.

In Matsuo algebras, Virasoro representations and the coset construction have particularly simple realisations. Results in Matsuo algebras lead us to conjecture a doubly-infinite family of vertex algebras related to a coset construction along the lines of Goddard-Kent-Olive.

Generalizing the Classical Spinning Particle

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Joint work with: Laurent Freidel

There is an abundance of models purporting to describe the classical spinning particle, but despite their variegation they are all constructed using a similar procedure. In particular, beginning with some initial phase space one performs symplectic reduction with respect to a constraint algebra which is selected to yield a reduced phase space of the appropriate dimension. All viable models must yield the same reduced phase space and so alternative descriptions should differ only superficially in the choice of initial phase space and corresponding constraint algebra. This, however, is not at all obvious and there seems to be no clear method for passing between competing models.

In the first half of this talk I address this difficulty by developing a dictionary relating various descriptions of the spinning particle. I begin by proposing a model in which the classical phase space is identified with the coadjoint orbits of the Poincare group. The advantage of this approach is two fold. First, it coincides with the quantum mechanical description of a spinning particle as an irreducible representation of the Poincare group. Second, it bypasses the need to perform symplectic reduction thereby eliminating much of the ambiguity inherent in other models. The dictionary is then developed by showing that the symplectic stucture on the coadjoint orbits can be mapped onto the symplectic structure of many other models via a judicious choice of coordinates.

Regardless of how the initial phase space is chosen, all models of the classical spinning particle include a constraint which is equivalent to the Mathisson–Pirani condition, i.e. $S_{ab}p^b = 0$ where S_{ab} is the spin angular momentum and p^b the linear momentum. This constraint, it turns out, is inevitable if one wishes the reduced phase space to coincide the coadjoint orbits of the Poincare group. In the second half of this talk I propose a generalization of the classical spinning particle in which this constraint is dropped. The model is obtained by considering the action of the Poincare group on the coadjoint orbits of the Poincare–Heisenberg group, given as $(3,1) \rtimes H(3,1)$ where $H(3,1)$ is the Heisenberg group. In this model a spinning particle is represented not as a point, but rather a ridged cylinder with a definite radius. Setting the radius to zero recovers the standard description of the spinning particle including the Mathisson–Pirani condition. While it won't be emphasised in the talk, I note that the proposed generalization of the spinning particle provides an intuitive notion of position; a feature absent from models based solely on the Poincare group.

Integrable boundary conditions and corresponding q-Knizhnik-Zamolodchikov equation.

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Joint work with: Jasper Stokman and Bart Vlaar

The talk will start with an overview of integrable boundary conditions in quantum field theory and statistical mechanics. The underlying algebraical structures are co-ideal subalgebras in corresponding Hopf algebras (quantum groups). One of its manifestations is the system of q-difference equations for correlation functions and formfactors known as boundary q-Knizhnik-Zamolodchikov systems. Integral formulae for solutions to these equations is a new result which will be presented at the end of the talk.

Quantum Confinement in Hydrogen Bond

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In this work, the quantum confinement effect is proposed as the cause of the displacement of the vibrational spectrum of molecular groups that involve Hydrogen-bonds. In this approach the H-bond imposes a space barrier to Hydrogen and constrains its oscillatory motion. We studied the vibrational transitions through the Morse potential, for the NH and OH molecular groups inside macromolecules in situation of confinement (when hydrogen bonding is formed) and non-confinement (when there is not hydrogen bonding). The eigenenergies are obtained through the variational method with the trial wave functions obtained from supersymmetric quantum mechanics (MQS) formalism. The results indicate that it is possible to distinguish by means of spectroscopy the adsortion/emission peaks related to the confined and non-confined cases. These results are satisfactorily compared with experimental results obtained from infrared spectroscopy.

Parabolic categories for bosonic ghost systems

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The longstanding problem of obtaining non-negative integer Verlinde coefficients for fractional level $\widehat{\mathfrak{sl}}(2)$ models has recently been resolved. One of the key points is that closure under the modular S-transformation requires representations which are not highest weight, but are only highest weight with respect to a parabolic subalgebra. Moreover, one needs to consider *all* proper parabolic subalgebras.

Motivated by the Wakimoto free field realisation of $\widehat{\mathfrak{sl}}(2)$, we study the analogous categories of representations for the $c = 2$ bosonic ghost system. Again, modular considerations prove that these parabolic highest weight modules must be present, confirming the conclusion reached, as a corollary of the fractional level analysis, for the $c = -1$ ghosts. We finish by discussing bosonic ghosts as examples of logarithmic conformal field theories.

On the Bergman spaces of slice hyperholomorphic functions

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Bergman theory admits two possible formulations for the class of slice hyperholomorphic functions. In the so-called Bergman theory of the first kind, we provide a Bergman kernel which is defined on Ω (which is a subset of $\mathbb{H}$ if we are considering slice regular functions of a quaternionic variable or it is a subset of the Euclidean space $\mathbb{R}^{n+1}$ in case we consider slice monogenic functions with values in a Clifford algebra) and is a reproducing kernel. In the so-called slice hyperholomorphic Bergman theory of the second kind, we use the Representation Formula to define another Bergman kernel; this time the kernel is still defined on Ω but it is a reproducing kernel via an integral computed on the intersection of Ω with suitable complex planes and the integral does not depend on the choice of the chosen complex plane. In this framework, we shall mainly discuss the formulation of the second kind and some of its properties. In particular, we show it is possible to write in closed form the Bergman kernel on the unit ball and on the half space of quaternions with real positive part.

Indecomposable representations of Temperley-Lieb algebras and their role in statistical physics

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Joint work with: Jonathan Belletête (U. de Montréal) and David Ridout (Australian Nat. U.)

The transfer matrix is a common tool in statistical physics and shares many properties with the Hamiltonian in quantum physics. However, contrarily to Hamiltonians, transfer matrices do not need to be Hermitian. They do not even need to have real eigenvalues. This distinction is of physical importance and efforts have been given to understand the representation theory of algebras arising in lattice models like, for example, the families of Temperley-Lieb algebras (TL algebras).

One telltale signature of this distinction is the existence of Jordan blocks in transfer matrices and, more generally, of indecomposable representations. I shall give a physical example of these Jordan blocks and report on efforts to classify and describe all indecomposable representations of the TL algebras and dilute TL algebras.

Neutral bions in the CP^{N-1} model

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Joint work with: Tatsuhiro Misumi and Muneto Nitta

We study classical configurations in the $\mathbb{C}P^{N-1}$ model on $\mathbb{R}^1 \times S^1$ with twisted boundary conditions. We focus on specific configurations composed of multiple fractionalized-instantons, termed "neutral bions", which are identified as "perturbative infrared renormalons" by Ünsal and his collaborators. For $\mathbb{Z}_N$ twisted boundary conditions, we consider an explicit ansatz corresponding to topologically trivial configurations containing one fractionalized instanton ($\nu = 1/N$) and one fractionalized anti-instanton ($\nu = -1/N$) at large separations, and exhibit the attractive interaction between the instanton constituents and how they behave at shorter separations. We show that the bosonic interaction potential between the constituents as a function of both the separation and N is consistent with the standard separated-instanton calculus even from short to large separations, which indicates that the ansatz enables us to study bions and the related physics for a wide range of separations. We also propose different bion ansatze in a certain non-$\mathbb{Z}_N$ twisted boundary condition corresponding to the "split" vacuum for $N = 3$ and its extensions for $N \geq 3$. We find that the interaction potential has qualitatively the same asymptotic behavior and N-dependence as those of bions for $\mathbb{Z}_N$ twisted boundary conditions.

On the nonlinear Fourier transform and its inverse

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We shall consider the nonlinear Fourier transform $\mathcal{F}$ associated with the periodic AKNS-ZS integrable systems. This transform is equivalent to the scattering transformation for the suitable integrable systems. The map $\mathcal{F}$ turns out to be an analytic map from the space $L^2[0, 2\pi]$ to the space $l^2_{\mathbb{Z}}$ of square-summable bi-infinite sequences. This fact enables us to construct a convergent iterative scheme by means of which we can calculate the inverse of $\mathcal{F}$ explicitly to any desired degree of accuracy.

Invariant metrics associated with the quaternionic Hardy space

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Joint work with: Nicola Arcozzi

The quaternionic Hardy space of slice regular functions is a reproducing kernel Hilbert space. In this talk we will see how this property can be exploited to construct a Riemannian metric on the quaternionic unit ball B and then we will discuss the geometry arising from this construction. We will also see that, in contrast with the example of the Poincaré metric on the complex unit disc, no Riemannian metric on B is invariant with respect to all slice regular automorphisms of B.

Surface defects and symmetries

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Joint work with: Jürgen Fuchs and Alessandro Valentino

Codimension-one defects in quantum field theories have proven to be natural objects that carry much interesting structure. In the talk, we concentrate on topological quantum field theories in three dimensions, with a special focus on Dijkgraaf-Witten theories with abelian gauge group. Surface defects in Dijkgraaf-Witten theories have applications in solid state physics, topological quantum computation and conformal field theory. We explain that symmetries in these topological field theories are naturally defined in terms of invertible topological surface defects and are thus Brauer-Picard groups.

Structure of the sets of mutually unbiased bases with cyclic symmetry

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Joint work with: Luis L. Sánchez-Soto and Gerd Leuchs

In the context of quantum state tomography, complete sets of mutually unbiased bases provide an attractive set of measurement bases as they achieve the minimal number of required different measurement setups. For qubit systems mutually unbiased bases can be constructed in a cyclic way which facilitates their implementation. In a recent article it has been shown that the entanglement structures of the bases play an important role concerning error distributions for certain properties of the physical system [1]. In order to find optimal cyclic sets subjected to this aspect, we will show how an existing construction scheme [2] can be generalized for this purpose. It turns out that the different sets have either a finite field, an additive group or an additive semigroup structure.

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Howe duality for polynomials on the superspace

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Joint work with: R. Lavicka, V. Soucek

We study representation theoretical aspects of harmonic analysis on the superspace $\mathbb{R}^{m|2n}$. The case of $m - 2n \in -2\mathbb{Z}_0$ is special due to the occurence of indecomposable, but not irreducible modules of both $\mathfrak{osp}(m|2n)$ and its Howe dual $\mathfrak{sl}(2, \mathbb{C})$. We present the multiplicity free decomposition of the space of $\mathbb{C}$-valued polynomials and some preliminary results in the spinor-valued case.

Momentum entanglement in relativistic quantum mechanics

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Within an irreducible unitary two-particle representation of the Poincaré group, the commutation relations of the Poincaré group require that the two-particle states are momentum entangled. As in gauge theories, momentum entanglement defines a correlation between the particles that can be described as an interaction provided by the exchange of virtual (gauge) quanta. The coupling constant of this interaction is uniquely determined by the structure of the irreducible two-particle state space. For two massive spin-half particles the coupling constant matches the empirical value of the electromagnetic coupling constant.

Classification and Identification of Lie Algebras

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Joint work with: Pavel Winternitz

In practical applications of Lie algebra theory the first task is usually to identify the algebra we are encountering as an abstract Lie algebra. We will review what we know about the classification and identification of Lie algebras, in particular solvable and Levi decomposable ones.

We shall start by an explicit example of one particular application of Lie group analysis in science – the identification of physically different problems formulated as PDEs which are found to be mathematically equivalent. Namely, we will consider shallow water equations with different shapes of the bottom and Coriolis force present or not.

After this motivational example we will review the general structure of Lie algebras, indicate which of their properties turn out to be most useful for the purpose of the identification of isomorphic algebras and introduce our recent results concerning the structure of solvable and Levi decomposable Lie algebras with the prescribed nilradical in an arbitrary finite dimension.

We will also highlight what the readers can (and cannot) find in our new monograph on the topic, scheduled to be published shortly before the conference:

Libor Šnobl and Pavel Winternitz: **Classification and Identification of Lie Algebras**, CRM Monograph Series, Volume: 33, American Mathematical Society together with Centre de Recherches Mathematiques, ISBN-13: 978-0-8218-4355-0.

The reduction of symmetry and special solutions in Clifford analysis

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In classical Clifford analysis the Dirac operator D acts on functions defined on Euclidean space and with values in the corresponding Clifford algebra C (or its complexification). However, it is possible to consider special solutions having values in a specific subspace V of C. The most interesting situation arises when the space V is invariant under a subgroup G of the symmetry group $\mathrm{Spin}(m)$ of the Dirac equation. In case G is a proper subgroup, the symmetry of the problem is reduced to G. This is very important because it leads directly to generalizations of Clifford analysis based on different symmetry groups.

More generally, if W is a vector space, it is possible to consider the twisted version D_W of the Dirac operator and to consider the same idea of symmetry reduction. The proposed construction leads to more explicit realizations of certain Stein-Weiss gradients and to their generalizations to more general symmetries. One of principal advantages of the construction is that special solutions of the (twisted) Dirac equation automatically inherit many properties of the solutions of the Dirac equation, the so-called monogenic functions.

Lie symmetry analysis and exact solutions of thermodiffusion equations

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The presentation deals with the study of equations describing thermodiffusion convection in a binary mixture via Lie symmetry approach. Thermodiffusion (Soret effect) is a molecular transport of substance associated with a thermal gradient. It results in component separation in non-uniformly heated fluid. Mathematical model of this process is described by Navier-Stokes equations and mass and heat transport equations [1]. Generally speaking mixture density and transport coefficients can not be only constants but also arbitrary functions of temperature and concentration [2].

Group classification problem for thermodiffusion equations is solved corresponding with above mentioned arbitrary functions [3]. Different specializations of the qualified functions and generators admitted by the governing equations depending on the values of these functions are found.

Some invariant and partially invariant solutions of the governing equations are constructed and applied to describe the thermodiffusion convection in the simple physical applications. Primarily we are interested in effects appearing under taking into account the nonconstant transport coefficients. It is very important for experimental work [4].

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Hermite polynomials and representations of the unitary group

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Joint work with: Diana Dziewa-Dawidczyk

Spaces of homogeneous complex polynomials in D variables form carrier spaces for representations of the unitary group $\mathbf{U}(D)$. These representations are well understood and their connections with certain families of classical orthogonal polynomials (Gegenbauer, Jacobi, and other) are widely studied. However, there is another realization for the action of the unitary group $\mathbf{U}(D)$ on polynomials, non necessarily homogeneous, in which Hermite polynomials in D variables play an important role. This action is related to the metaplectic (oscillator) representation, and was studied some time ago be the present author and, independently, by A. Wünsche for $D = 2$. In this talk we want to concentrate on this second realization and describe its properties in a more comprehensive way, as well as point out connections with the orthogonal polynomials, notably Jacobi. Some links to the coherent states for the $\mathbf{SU}(D)$ groups are also presented.

Softening the edges of the Lax-Phillips scattering theory for quantum mechanics

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Let $\mathcal{H}$ be a separable Hilbert space, $\{U(t)\}_{t\in\mathbb{R}}$ a unitary evolution group defined on $\mathcal{H}$ and $\mathcal{D}_\pm$ two closed subspaces of $\mathcal{H}$ such that:

(i) $\mathcal{D}_- \perp \mathcal{D}_+$, (ii) $U(t)\mathcal{D}_- \subseteq \mathcal{D}_-$ for $t \le 0$, (iii) $U(t)\mathcal{D}_+ \subseteq \mathcal{D}_+$ for $t \ge 0$,

(iv) $\cap_{t\in\mathbb{R}} U(t)\mathcal{D}_\pm = \{0\}$, (v) $\overline{\vee_{t\in\mathbb{R}} U(t)\mathcal{D}_\pm} = \mathcal{H}$.

The subspaces $\mathcal{D}_-$ and $\mathcal{D}_+$ are called, respectively, the *incoming subspace* and *outgoing subspace* for $U(t)$. The Lax-Phillips scattering theory applies for any scattering problem for which conditions (i)-(v) hold. It has been originally devised for the description of scattering of electromagnetic or acoustic waves, governed by linear hyperbolic wave equations, off compactly supported (star shaped) obstacles. The Hilbert space $\mathcal{H}$ contains all scattering states, the evolution group $\{U(t)\}_{t\in\mathbb{R}}$ describes the scattering of a (finite energy) wave packet off the obstacle and the incoming and outgoing subspaces $\mathcal{D}_-$ and $\mathcal{D}_+$ represent wave packets that do not interact with the obstacle for $t \le 0$, respectively $t \ge 0$. The existence of the subspaces $\mathcal{D}_\pm$ is natural for the hyperbolic wave equation (Huygens' principle) and their sharp stability properties are essential for the implementation of the Lax-Phillips structure. Lax and Phillips define a family of objects (describing wave packet - obstacle interaction) $\{Z^{\text{LP}}(t)\}_{t\in\mathbb{R}^+}$,

$$Z^{\text{LP}}(t) := P_{\mathcal{K}} U(t) P_{\mathcal{K}}, \quad t \ge 0,$$

with $\mathcal{K} = \mathcal{H} \ominus (\mathcal{D}_- \oplus \mathcal{D}_+)$, which is known as the Lax-Phillips semigroup. The main result of the Lax-Phillips theory states that resonance poles of the (Lax-Phillips) S-matrix correspond to eigenvalues of the generator of the Lax-Phillips semigroup and the corresponding eigenstates are the resonance states.

The structure of the Lax-Phillips scattering theory is appealing since resonance poles are directly associated with eigenvalues and eigenstates of a dynamical semigroup. However, attempts to use it in the context of quantum mechanics have encountered considerable difficulties over the years since the basic assumptions of the theory cannot be satisfied, in general, by quantum mechanical scattering problems. I shall describe a recently developed method of relaxing the assumptions of the Lax-Phillips theory such that the resulting "softer" version of the theory is applicable to quantum mechanical scattering.

Spontaneous supersymmetry breaking and instanton sum in 2D type IIA superstring theory

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Joint work with: M. G. Endres, T. Kuroki, S. M. Nishigaki, H. Suzuki

We consider a double-well supersymmetric matrix model and its interpretation as a nonperturbative definition of two-dimensional type IIA superstring theory. The interpretation is confirmed by direct comparison of symmetries and amplitudes in both sides of the matrix model and the string theory [1].
We show that instanton contributions in the matrix model survive in the double scaling limit and induce spontaneous supersymmetry breaking [2]. It implies that the target-space supersymmetry is spontaneously broken due to nonperturbative effects in the string theory.
Finally, we obtain the full nonperturbative free energy in terms of the Tracy-Widom distribution in random matrix theory [3].

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The method of Riemann-Hilbert problems for 3D-Ising model

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The Riemann-Hilbert problem in several complex variables is presented and exact solutions for 3D-Ising model are constructed by use of the method. The outline is described as follows: (1) The geometrical representation of solutions of 3D-Ising model is presented, which can be described in terms of knots. (2) The knots are described on hyperelliptic Riemann surfaces and the monoddromy representation of the geometric representation is constructed. (3) The Riemann-Hilbert problem for the monodromy representation is formulated and solutions are constructed.

Bessel-to-Parseval via Julia dilations

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This is an attempt at revitalizing the seventy year old dilation ideas of Julia, prompted by the contemporary development of the theory of frames with some connection with related objects.

Comments on supersymmetric quantum field theories

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Supersymmetric quantum field theories (SUSY QFTs) have been studied intensively for the last few decades. Originally, the main task was to study the properties of individual SUSY QFTs. Gradually, it came to be realized that two SUSY QFTs constructed in two distinct manners can describe the same SUSY QFT: an early example is the celebrated mirror symmetry and in general such equivalences are called dualities.

In the last few years, we are becoming increasingly aware that there are more structures among SUSY QFTs, not just dualities relating just two SUSY QFTs. For example, a large class of SUSY QFTs in four dimensions can be naturally mapped to a two-dimensional QFT, such that the structures among this class of 4d SUSY QFTs are mapped to the structures among observables in this particular 2d QFT. There seems to be a certain shift in the emphasis from the study of individual SUSY QFTs to the study of the entirety of all possible SUSY QFTs and their interrelationship.

In this talk, I would like to present to a wider mathematical-physics community what this new viewpoint can tell us about. I will use the 'category' of four-dimensional $\mathcal{N}=2$ supersymmetric quantum field theories as a primary example, discuss some of its internal structures, and discuss how these structures can be mapped wholesale into the structures among better known mathematical-physical objects. The most interesting case is when the structures thus obtained are hitherto-unknown ones among known mathematical-physical objects, and some concrete examples will be discussed.

Stability analysis for the free rigid body on $U(n)$

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Joint work with: Tudor Ratiu

A natural extension of the free rigid body dynamics to the unitary group $U(n)$ is considered. The dynamics is described by the Euler equation on the Lie algebra $\mathfrak{u}(n)$, which has a bi-Hamiltonian structure, and it can be reduced onto the adjoint orbits, as in the case of the $SO(n)$. The complete integrability and the stability of the equilibria on the generic orbits are considered by using the method of Bolsinov and Oshemkov. In particular, it is shown that all the equilibria on generic orbits are Lyapunov stable.

Formal Groups, Number Theory and Entropies

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Formal group theory [1] plays a central role in modern algebraic topology and mathematical physics. It will be shown that universal Bernoulli polynomials and a class of L-functions can be associated with the Lazard universal formal group [2]. General Almkvist-Meurman-type congruences and summation formulae are constructed [5]. At the same time, nonadditive entropies, relevant in many contexts of non-equilibrium thermodynamics can be defined from realizations of the multiplicative, Euler and Abel formal groups, as well as infinitely many new cases of entropic functionals and Kullback-Leibler divergences [3],[6]. We also mention that integrable maps can be obtained from an algebraic approach related to formal groups [4]. The proposed construction paves the way to the study of new connections among group theory, special functions, statistical mechanics and number theory.

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Finite temperature Casimir interaction between spheres in higher dimensional spacetime

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We consider the finite temperature Casimir interaction between two spheres in (D+1)-dimensional Minkowski spacetime subject to the vacuum fluctuations of a scalar field with Dirichlet or Neumann boundary conditions. Our main objective is to derive the Casimir free interaction energy. First, we have to express the waves in hyper-spherical coordinates about the two spheres respectively. Multiple scattering formalism is then used to relate the different representations of the waves. Using the fact that the hyper-spherical harmonics can be expressed in terms of homogeneous polynomials, we define an operator that can be used to generate hyper-spherical waves of higher wave numbers from the hyper-spherical waves of lowest wave number. This allows us to compute the translation matrices that relate the two hyper-spherical coordinate systems. From these, we can write down the dispersion relation for the eigen-frequencies of the system. Zeta function and contour integration techniques are then used to derive the Casimir free energy.

Finite quantum kinematics and their symmetries

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Joint work with: M. Korbelar

Quantum mechanics in Hilbert spaces of finite dimension N is studied from the point of view.of inequivalent quantum kinematics. The fundamental theorem describing all finite discrete abelian groups of order N as direct products of cyclic groups, whose orders are powers of not necessarily distinct primes contained in the prime decomposition of N, leads to a classification of inequivalent finite Weyl systems and finite Weyl-Heisenberg groups. The representation theoretic approach is then confronted with the fine gradings of the operator algebra. The relation between the mathematical formalism and physical realizations of finite quantum systems is discussed from this perspective. The corresponding symmetries - normalizers of the Weyl-Heisenberg groups in unitary groups (in quantum information conventionally called Clifford groups) - are fully described. Special attention is devoted to symmetries of the elementary building blocks of finite quantum systems (quantal degrees of freedom).

Four types of superconformal mechanics: D-module reps and invariant actions

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Joint work with: N. L. Holanda

(Super)conformal mechanics in one dimension is induced by parabolic or hyperbolic/trigonometric transformations, either homogeneous (for a scaling dimension λ) or inhomogeneous (at $\lambda = 0$, with ρ an inhomogeneity parameter). Four types of inequivalent (super)conformal actions are thus obtained. With the exclusion of the homogeneous parabolic case, dimensional constants are present.

Both the inhomogeneity and the insertion of λ generalize the construction of Papadopoulos [CQG 30 (2013) 075018; arXiv:1210.1719].

Inhomogeneous D-module reps are presented for the $d = 1$ superconformal algebras $osp(1|2)$, $sl(2|1)$, $B(1,1)$ and $A(1,1)$. For centerless superVirasoro algebras D-module reps are presented (in the homogeneous case for $\mathcal{N} = 1,2,3,4$; in the inhomogeneous case for $\mathcal{N} = 1,2,3$).

The four types of $d = 1$ superconformal actions are derived for $\mathcal{N} = 1,2,4$ systems. When $\mathcal{N} = 4$, the homogeneously-induced actions are $D(2,1;\alpha)$-invariant (α is critically linked to λ); the inhomogeneously-induced actions are $A(1,1)$-invariant.

In $d = 2$, for a single bosonic field, the homogeneous transformations induce a conformally invariant power-law action, while the inhomogeneous transformations induce the conformally invariant Liouville action.

Multihyperbolic Analysis

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Joint work with: Adrian Vajiac

The analysis of the space of hyperbolic numbers has gained momentum in the past decade or so in the literature. Even if the hyperbolic space is a degenerate case in Clifford Analysis, it holds strong interest for the physics of special relativity. In this talk we introduce the analysis of multihyperbolic spaces which are built recursively from the hyperbolic algebra. In particular, we focus on the study of differential operators in this context.

Group foliation of finite difference equations

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Joint work with: Robert Thompson

The method of group foliation developed by Vessiot (also called group splitting, or group stratification) is a powerful procedure for obtaining invariant, partially invariant and non-invariant solutions of differential equations invariant under a symmetry group. Using the theory of equivariant moving frames we extend the method of group foliation to finite difference equations. The procedure yields (non-invariant) solutions of finite difference equations invariant under a finite-dimensional symmetry group. The method will be illustrated with several examples.

Gauge-covariant extensions of Killing tensors and conservation laws

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In classical and quantum mechanical systems on manifolds with gauge-field fluxes, constants of motion are constructed from gauge-covariant extensions of Killing vectors and tensors. This construction can be carried out using a manifestly covariant procedure, in terms of covariant phase space with a covariant generalization of the Poisson brackets, c.q. quantum commutators. Some examples of this construction are presented.

Eigenfunctions of the higher spin operator $\mathcal{R}_k^s$ on S^{m-1}

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Let $\mathcal{H}_k(\mathbb{R}^m, V)$ and $\mathcal{M}_k(\mathbb{R}^m, V)$ be the spaces of V-valued spherical harmonics and monogenics of order k on $\mathbb{R}^m$. To fix ideas, take m odd. By choosing $V = \mathbb{C}$ or a spinor space $\mathbb{S}$, these spaces are concrete realizations of the irreducible representations of Spin(m) with weight $(k, 0, \ldots, 0)$ and $(k + \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2})$. On $S^{m-1} \times \mathcal{H}_k(\mathbb{R}^m, V)$ and $S^{m-1} \times \mathcal{M}_k(\mathbb{R}^m, V)$ we define the bundle constraint:

$$\omega \sim P \ \text{ if } \ \langle \omega, \partial_u \rangle P(u) = 0 \,.$$

The $\sim$ quotient gives rise to the bundles $\widetilde{\mathcal{H}_k}(S^{m-1}, V)$ and $\widetilde{\mathcal{M}_k}(S^{m-1}, V)$. Sections of these bundles are identified with functions $f(\omega, u)$ which for fixed $\omega \in S^{m-1}$ belong to either $\mathcal{H}_k(\mathbb{R}^m, V)$ or $\mathcal{M}_k(\mathbb{R}^m, V)$ and such that $\langle \omega, \partial_u \rangle f(\omega, u) = 0$.
The twisted Dirac operator $\nabla^s := \omega(\frac{m-1}{2} - \Gamma_\omega)$ on S^{m-1} respects the bundle constraint, hence

$$\nabla^s : C^\infty(\widetilde{\mathcal{M}_k}(S^{m-1}, V)) \to C^\infty(\widetilde{\mathcal{H}_k}(S^{m-1}, V)).$$

Take a fixed $\omega \in S^{m-1}$ and consider the action of a tangent vector a in ω on $S_k \in \mathcal{M}_k(\omega^\perp, V)$. Using the Fischer decomposition in $\mathbb{R}^{m-1} \simeq \omega^\perp$,

$$aS_k \in \mathcal{H}_k(\omega^\perp, V) = \mathcal{M}_k(\omega^\perp, V) \oplus \omega(\omega \wedge u) \cdot \mathcal{M}_{k-1}(\omega^\perp, V).$$

The projection on the first summand of this decomposition is denoted by $\pi^k_{k,\omega}$. Let $f(\omega, u) \in C^\infty(\widetilde{\mathcal{M}_k}(S^{m-1}, V))$. The higher spin operator on S^{m-1} associated to the bundle $\widetilde{\mathcal{M}_k}(S^{m-1}, V)$ is the conformally invariant operator

$$\mathcal{R}_k^s : C^\infty(\widetilde{\mathcal{M}_k}(S^{m-1}, V)) \to C^\infty(\widetilde{\mathcal{M}_k}(S^{m-1}, V))$$

defined as

$$\mathcal{R}_k^s f(\omega, u) := \pi^k_{k,\omega} \nabla^s f(\omega, u) \,.$$

These operators ($k \in \mathbb{N}$) are examples of generalized gradients or Stein-Weiss operators on S^{m-1}. The invariance property of $\mathcal{R}_k^s$ fixes the spectrum of $\mathcal{R}_k^s$ up to a constant factor. This follows from abstract representation theoretical arguments (cf. the work of Branson, Olafsson and Orsted, on spectrum generating operators). Here we obtain the spectrum in a completely different way. At the same time we also construct eigenfunctions for $\mathcal{R}_k^s$ in terms of highest weight vectors for the action of Spin(m).

Exploring the phase diagram of the px + ipy pairing Hamiltonian by linking the eigenstates to associated bosonic states.

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Joint work with: Stijn De Baerdemacker and Dimitri Van Neck

Recently, interest has increased in the hyperbolic family of integrable Richardson-Gaudin (RG) models. It was pointed out that a particular linear combination of the integrals of motion of the hyperbolic RG model leads to a Hamiltonian that describes p-wave pairing in a two dimensional system[1]. Such an interaction is found to be present in fermionic superfluids (^{3}He), ultra-cold atomic gases and p-wave superconductivity. Furthermore the phase diagram is intriguing, with the presence of the Moore-Read and Read-Green lines. At the Read-Green line a rare third-order quantum phase transition occurs. We used a bosonization technique[2, 3] to exactly solve the $p_x + ip_y$ pairing Hamiltonian. This made it possible to investigate the effects of the Pauli principle on the energy spectrum, by gradually reintroducing the Pauli principle. It also introduces an efficient and stable numerical method to probe all the eigenstates of this class of Hamiltonians. The bosonic states introduce a labeling of the important eigenstates of the $p_x + ip_y$ pairing Hamiltonian. Patterns were found which link bosonic states to the important states of the $p_x + ip_y$ pairing Hamiltonian at the Moore-Read and Read-Green lines. The circumvention of critical points, where two or more RG variables become equal, is therefore not necessary, and an efficient investigation of the interesting points of the phase diagram is possible.

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The Bannai-Ito algebra and some applications

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Joint work with: H. de Bie (Ghent), V.X. Genest (Montreal), S. Tsujimoto (Kyoto), A. Zhedanov (Donetsk)

The Bannai-Ito algebra will be presented together with some applications. Its relation with the Bannai-Ito polynomials, the Racah problem for the paraboson algebra, a superintegrable model with reflections as well as the Dirac-Dunkl equation on the 2-sphere will be surveyed.

Quantum probabilities versus Kolmogorov probabilities and Bell inequalities

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The orthomodular lattice L[H(d)] of a quantum system with d-dimensional Hilbert space H(d) is studied. Sublattices which are Boolean algebras play the role of contexts. They are explicitly constructed for the case d = 4 and they are used in the study of CHSH inequalities for a system comprised of two spin 1/2 particles. The generalized additivity relation $q(A \vee B) - q(A) - q(B) + q(A \wedge B) = 0$ which holds for Kolmogorov probabilities, and also the Boole inequality $q(A \vee B) \leq q(A) + q(B)$ which follows from it, are violated by quantum probabilities in the full lattice L[H(d)] (they are only valid within the Boolean algebras). The violation of CHSH inequalities is shown to be intimately related to this violation of the Boole inequality.

Lifted tensors and Hamilton-Jacobi separability

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Joint work with: W. Sarlet

We will discuss natural lifting operations from a bundle $\tau : E \to \mathbb{R}$ to the dual of the first-jet bundle, which is the appropriate manifold for the geometric description of time-dependent Hamiltonian systems. The main purpose is to define a complete lift of a type $(1, 1)$ tensor field on E. This construction and its properties are, in particular, relevant for an intrinsic characterization of Forbat's conditions for separability of the time-dependent Hamilton-Jacobi equation.

Fluctuations of TASEP and LPP with general initial data

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Joint work with: Ivan Corwin and Zhipeng Liu

The totally asymmetric simple exclusion process (TASEP) is an integrable model in the Kardar–Parisi–Zhang (KPZ) universality class. It has a correspondence with the last-passage percolation (LPP) model. The asymptotic properties of TASEP have been studied extensively with special initial data (e.g., step, flat, Bernoulli, etc). In this talk we present a limiting formula for the point-to-line LPP where the line is a curve of general shape, and then get a limiting formula for the fluctuation of a particle in TASEP with general initial data. Our limiting formula is expressed in the Airy process and it is derived with the help of a uniform slow decorrelation property of the LPP. Our general formulas have interesting consequences with special initial data.

Representations of sl(2,C) in the BGG Category O and master symmetries

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In this talk, we first recall the structure of the indecomposable $\mathfrak{sl}(2,)$ modules in the Bernstein-Gelfand-Gelfand (BGG) category $\mathcal{O}$. We show these modules naturally arise for homogeneous integrable systems. We then develop an approach to construct local master symmetries. This naturally includes the hierarchy of time-dependent symmetries. We illustrate the method by classical and new examples.

On the Hopf algebra structures of generalized Radford's biproducts

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In this paper, we give a generlized structure of Radford's biproduct, Majid's double cross product and unified product, and we give the necessary and sufficient conditions of $A \ltimes H$ to be a Hopf algebra. At last, some results related to this structure is presented.

On Mathieu Moonshine

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We present some recent developments in Mathieu Moonshine, which links the sporadic Mathieu group M_{24} to the elliptic genus of those superconformal field theories that are associated to K3 surfaces.

Position and momentum in the monochromatic Maxwell fish-eye

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Joint work with: Cristina Salto-Alegre

In geometric optics the Maxwell fish-eye is a medium where light rays follow circles, while in scalar wave optics this medium can only 'trap' fields of certain discrete frequencies. In the monochromatic case characterized by a positive integer ℓ, there are $2\ell + 1$ independent fields. We identify two bases of functions that serve as wavefields of definite position and definite momentum. Their construction uses the stereographic projection of the sphere, and the identification is corroborated in the $\ell \to \infty$ limit to a homogeneous Helmholtz medium.

Symmetric coordinates in solids. Magnetic Bloch oscillations.

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There has recently been a revival of the Bloch theory for energy bands in solids. This revival was caused, on one hand, by the discovery of topological insulators and the discovery of graphene, and, on the other end , by a very efficient new technique that was developed for creating artificial solids. These are the cold atoms in optical lattices. Last year geometric phases were measured in energy bands of cold atoms in a one-dimensional optical lattice by using Bloch oscillations. These phases are related to the Wyckoff positions, or the symmetry centers in the Bravais lattice. In this lecture a theoretical frame is developed for magnetic Bloch oscillations, meaning oscillations in the presence of a magnetic field. The theory is based on the kq-representation and the symmetric coordinates in solids. It is shown that for a Bloch electron in a magnetic field the orbit quasi-center is a conserved quantity. As is well known, in the absence of the periodic potential, the orbit center itself is a conserved quantity. This is similar to the conservation of the quasi-momentum for an electron in a periodic potential. In the absence of the periodic potential, the momentum itself is a conserved quantity. When an electric field is turned on, the orbit quasi-center oscillates in a similar way to the Bloch oscillations in the absence of a magnetic field. But there are some differences. When there is no magnetic field the symmetry of the problem is the translation group. In the presence of a magnetic field, the symmetry of the problem is the magnetic translation group, with the magnetic Brillouin zone depending on the strength of the magnetic field. An analogy is drawn between Bloch oscillations and magnetic Bloch oscillations. By using the magnetic translations it is indicated that a magnetic Wannier-Stark ladder appears in the spectrum of a Bloch electron in crossed magnetic and electric fields. The geometric phases for magnetic Bloch oscillations and the width of the magnetic Wannier-Stark levels should be magnetic field dependent.

Beltrami-Courant Differentials and Homotopy Gerstenhaber algebras

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I will talk about the homotopy Gerstenhaber algebras describing the symmetries of 2d first order sigma models and their relation to the structure of Einstein equations.

Braided autoequivalences and quantum commutative bi-Galois objects

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Joint work with: Haixing Zhu

Let (H, R) be a quasitriangular weak Hopf algebra over a field k. We show that there is a braided monoidal equivalence between the Yetter-Drinfeld module category ${}_H^H YD$ over H and the category of comodules over some braided Hopf algebra ${}_R H$ in the category ${}_H M$. Based on this equivalence, we prove that every braided bi-Galois object A over the braided Hopf algebra ${}_R H$ defines a braided autoequivalence of the category ${}_H^H YD$ if and only if A is quantum commutative. In case H is semisimple over an algebraically closed field, i.e. the fusion case, then every braided autoequivalence of ${}_H^H YD$ trivializable on ${}_H M$ is determined by such a quantum commutative Galois object. The quantum commutative Galois objects in ${}_H M$ form a group measuring the Brauer group of (H, R) like in the Hopf algebra case.

A Microscopic Theory of Neutron

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We present a microscopic theory of the neutron in a framework consistent with the Standard Model in respect both to the empirical laws and quantum field theoretic description. The theory consists in a model neutron constructed using relevant experimental observations as input information, and predictions of the basic properties of neutron based on first principles solutions. The neutron is proposed to be composed of an electron e and a proton p which are separated at a distance $r_1 \sim 10^{-18}$ m, and in highly relativistic angular motions distinctly along an orbit of radius r_1 of the $l = 1$ eigenstate about their common instantaneous centre of mass. The associated electrically neutral vortex entity has an angular momentum $(1/2)\hbar$ and is identifiable as a confined antineutrino $\bar{\nu}_e$. At the scale $r_1 \sim 10^{-18}$ m, the e, p particles are attracted with one another predominantly by a central magnetic force as result of the particles' relative orbital and spin motions. The interaction force (resembling the weak force), potential (resembling the Higgs' field), and accordingly Hamiltonian (H_1), as well as the intermediate vector boson (W^+, W^-, Z^0) masses, are derived based directly on first principles laws in a unified framework of electromagnetism, quantum mechanics, and relativity. Especially, we predict the equation of $-\mathsf{H}_1 r_1^3$, which is directly comparable with the Fermi constant G_F, as $-\mathsf{H}_1 r_1^3 = G_F = \frac{A_1(1-C_{k,1})}{3r_1^3}$, where $A_1 = \frac{\sqrt{g_p g_e} e^2 \hbar^2 C}{8\pi\epsilon_0 \gamma m_e^0 m_p^0 c^2}$; m_e^0, m_p^0 are the electron and proton rest masses, γ is a Lorentz factor, $C_{k,1}, C$ are kinetic and geometric factors, and the remainder ($g_p, g_e, e, \hbar, \epsilon_0, c$) the usual fundamental constants. With the experimental value for $G_F (= 1.435 \times 10^{-62}$ Jm^3) as a numerical constraint on $-\mathsf{H}_1 r_1^3$, unique numerical solution for a (meta) stationary neutron is found to exist at an extremal point $r_1 = 2.() \times 10^{-18}$ m such that $|\mathsf{H}_1 r_1^3|$ is a minimum, and hence the neutron lifetime ($\propto |\mathsf{H}_1 r_1^3|^{-1}$) is a maximum. First principles solutions and predictions obtained also include the neutron spin (1/2) and (apparent) magnetic moment, the weak mixing (or Weinberg) angle θ_w between the charged and neutral vector boson masses, the β-decay reaction equation, and the V,A symmetries of the lepton emission directions versus spin-vectors and the parity pertaining to β-decay, that are in overall agreement with experiment. The first-principles microscopic description of neutron may be furthermore phenomenologically meaningfully mapped on to an isospin–"effective charge" representation with the $SU(2) \times U(1)$ symmetry of the GWS electroweak theory. The microscopic theory has furthermore a built-in scheme for the two other generations of leptons μ, τ and (anti)neutrinos, the other neutron-like systems such as Λ, Σ, and more generally the other elementary particles involving weak interactions.

Properties of the series solutions for Painlevé transcendents: from P_{IV} to P_{I} through P_{XXXIV}

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Joint work with: A.N.W. Hone

The Hamiltonian functions for the Painlevé equations are non autonomous polynomials in the phase space variables. As functions of time, they satisfy certain differential equations and are usually called σ functions. Here we show how a multi parametric Chazy-type equation encompasses the σ functions for the Painlevé equations I, II, XXXIV and IV. The Mittag-Leffler expansion for the solutions of the Chazy-type equation and the asymptotic behaviour of the coefficients of its Laurent series expansion are presented. These results extend analogous formulae for the case of the Painlevé I equation given in a previous work [*A.N.W. Hone, O. Ragnisco and F. Zullo: JNMP, 20, Sup. 1, 2013*]. Thanks to a one-to-one correspondence among the σ functions and the solutions of the corresponding Painlevé equations, the Mittag-Leffler expansions and the asymptotic behaviour of the coefficients of the Laurent series expansions for Painlevé IV, XXXIV and Painelvé I are discussed. Explicit quadratic recurrence formulae for the Taylor series of the respective tau-functions around a zero are also given. Finally numerical and exact results on the symmetric solutions singular at the origin are presented.

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